Appendix B: Catalogue of minimal asymmetric graphs with iteratively removed vertices

The aim of this bachelor thesis is to measure the degree of asymmetry of an asymmetric graph by the number of vertices which we have to delete to obtain a symmetric graph.

This catalogue shows all minimal asymmetric graphs 3.11 and graphs, from which we iteratively remove vertices to make them symmetric. We obtained these graphs using the GAP program and their visual display using the JupyterViz package. Since the original asymmetric graphs are minimal, each new graph obtained after gradual deletion of vertices has an automorphism group of at least two. All symmetric graphs obtained by removing the same number of vertices from the minimal asymmetric graphs are non-isomorphic to each other. Non-isomorphic graphs may appear in different groups after removing a different number of vertices, which is consistent with our proposal. This helps the program to render better and has a visually better system. Using the GAP program and its GRAPE package, we obtained the automorphism group for each symmetric graph.

3.1 Minimal asymmetric graphs

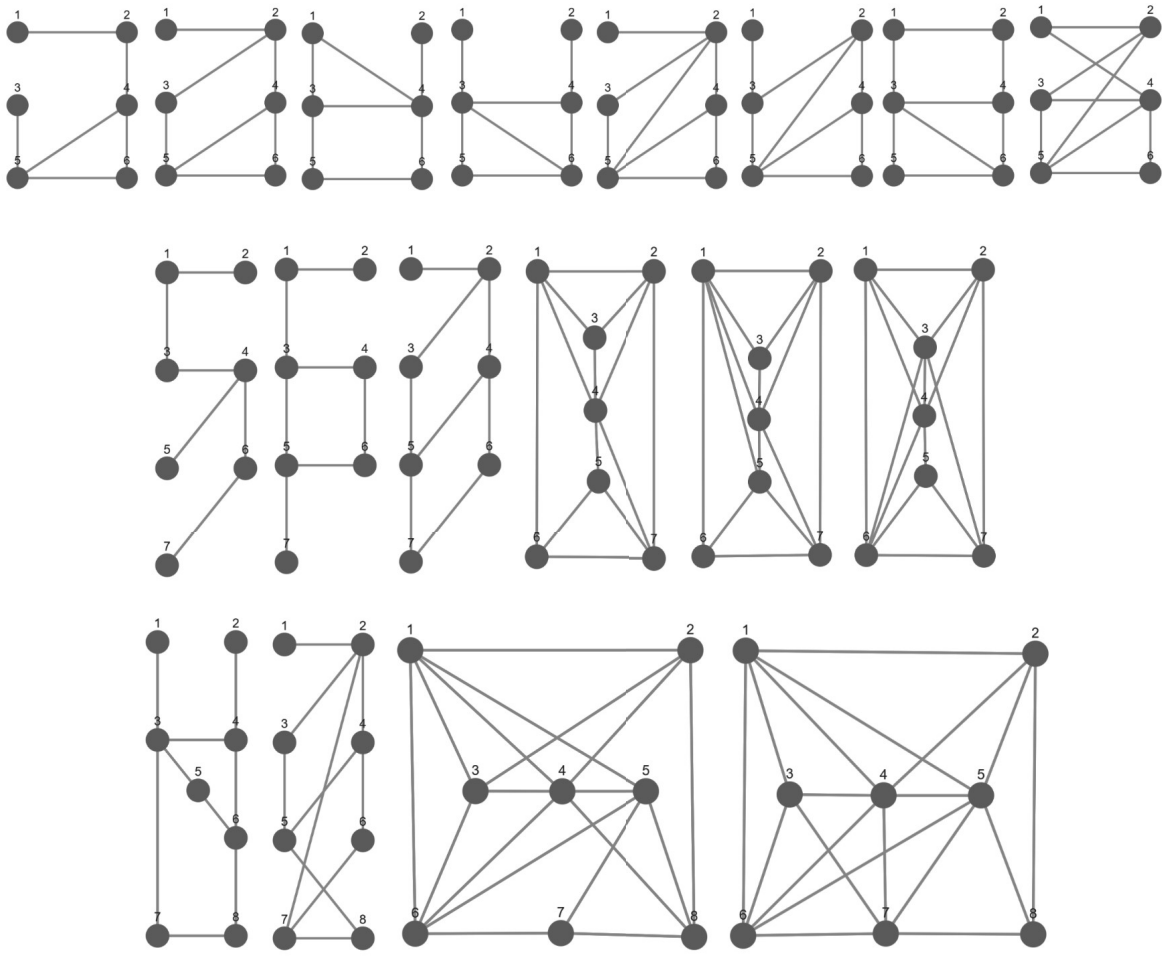

Figure 3.11: Final list of 18 minimal asymmetric graphs. [6]

3.2 Minimal asymmetric graphs without one vertex

Minimal asymmetric graphs without one vertex have an automorphism group mostly of size 2. Using our program, we found forty-four different graphs, which we obtained by deleting one vertex from minimal asymmetric graphs. Of these, nine graphs have an automorphism group of size 4. The other ones have an automorphism group of size 2.

3.2.1 Graphs with an automorphism group of size 2

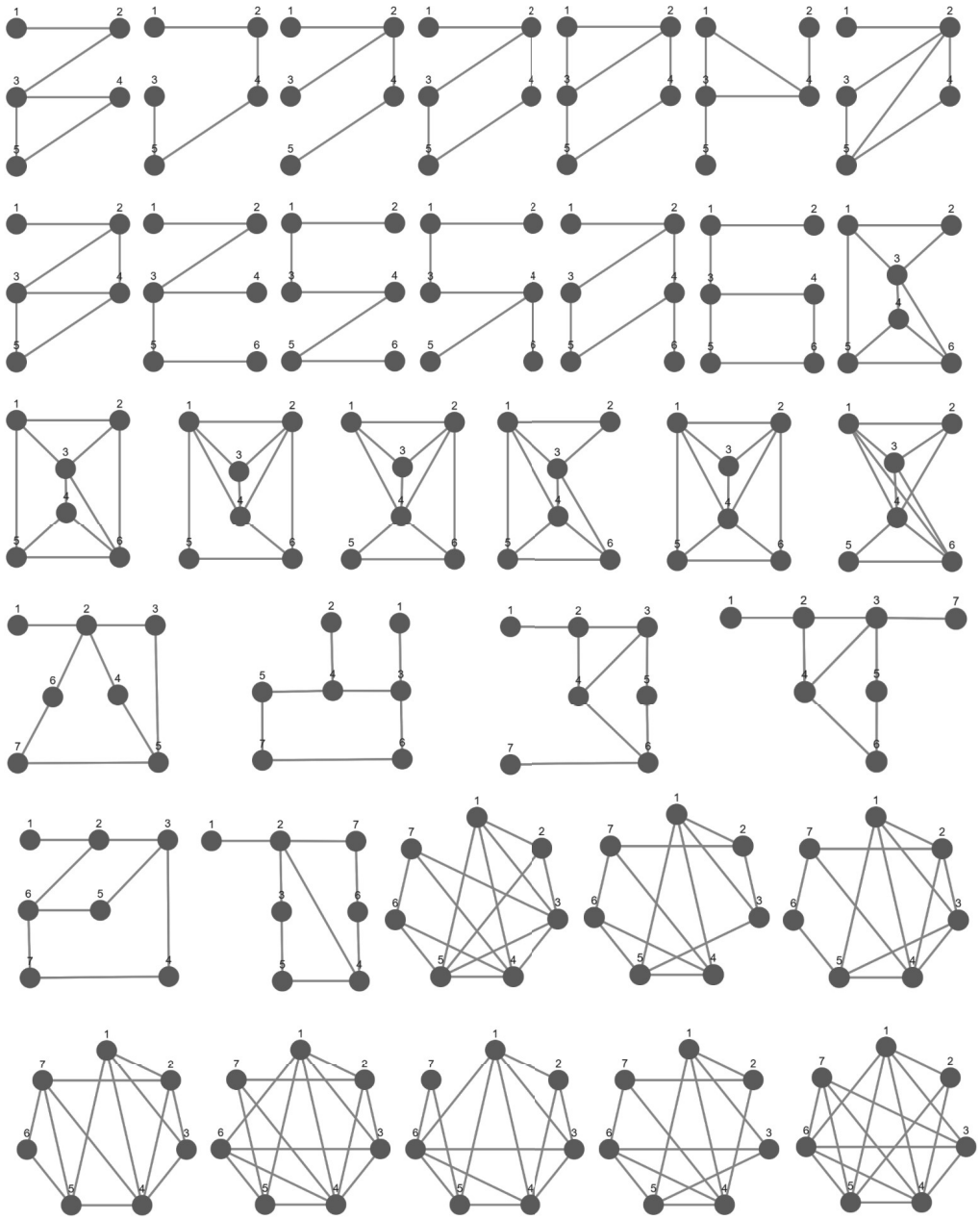

Figure 3.12: List of minimal asymmetric graphs without one vertex with an automorphism group of size 2

3.2.2 Graphs with an automorphism group of size 4

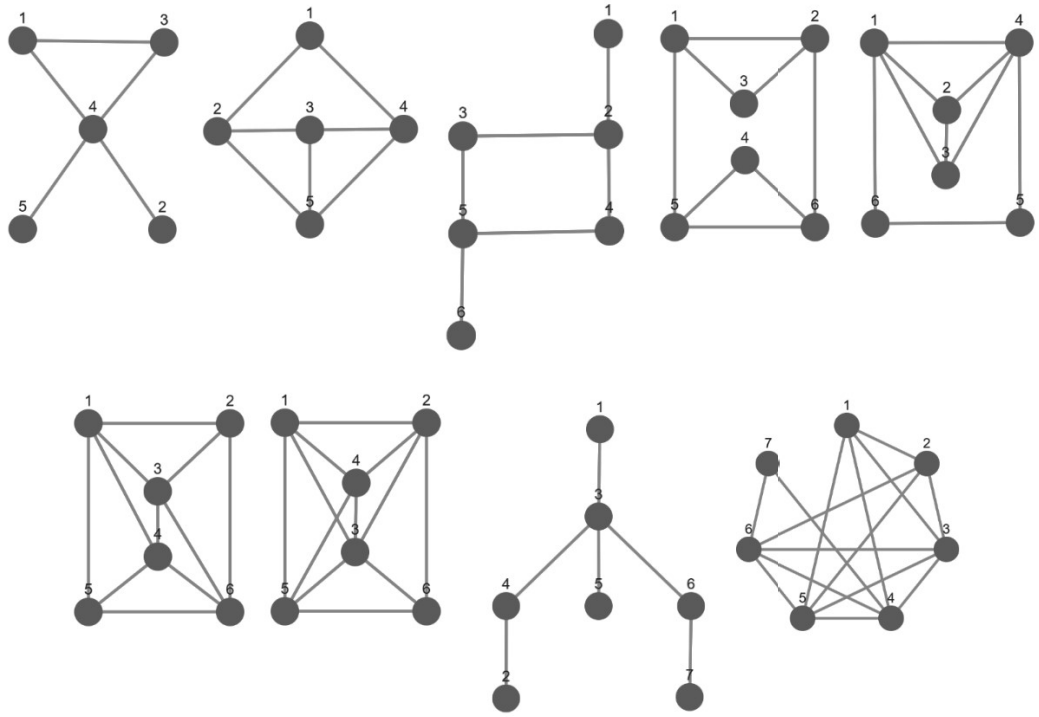

Figure 3.13: List of minimal asymmetric graphs without one vertex with an automorphism group of size 4

3.3 Minimal asymmetric graphs without two vertices

There are thirty-one connected graphs, that we can obtain from minimal asymmetric graphs, when we delete three vertices from all of them. They have an automorphism group size of 2, 4, 6, 8, 12, or 16.

3.3.1 Graphs with an automorphism group of size 2

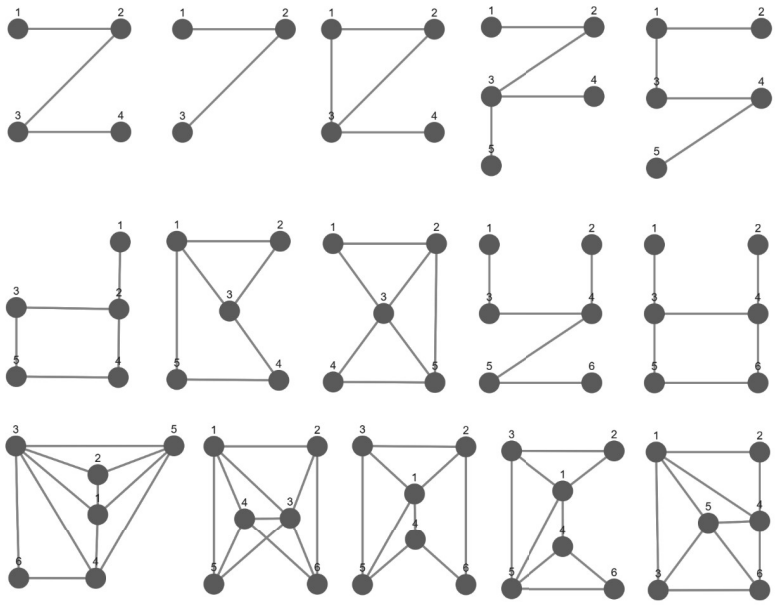

Figure 3.14: List of minimal asymmetric graphs without two vertices with an automorphism group of size 2

3.3.2 Graphs with an automorphism group of size 4

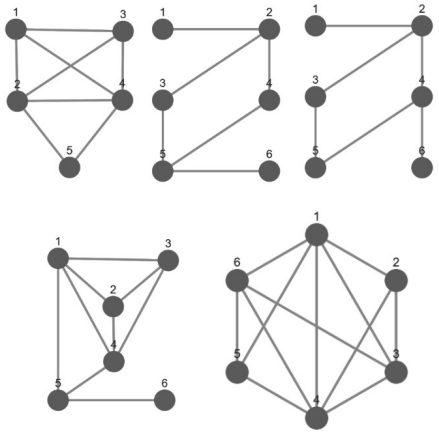

Figure 3.15: List of minimal asymmetric graphs without two vertices with an automorphism group of size 4

3.3.3 Graphs with an automorphism group of size 6

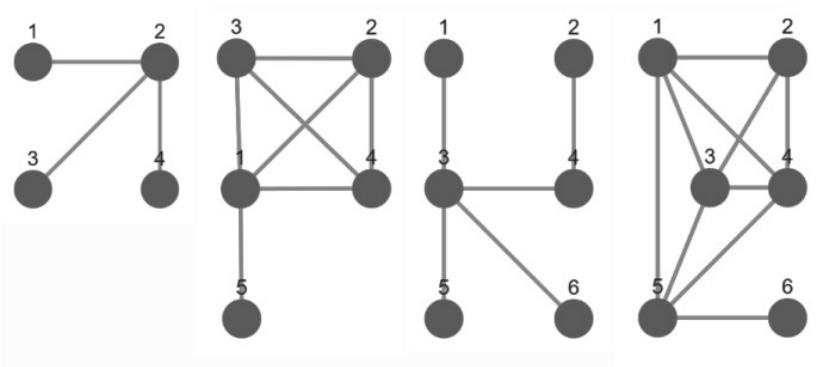

Figure 3.16: List of minimal asymmetric graphs without two vertices with an automorphism group of size 6

3.3.4 Graphs with an automorphism group of size 8

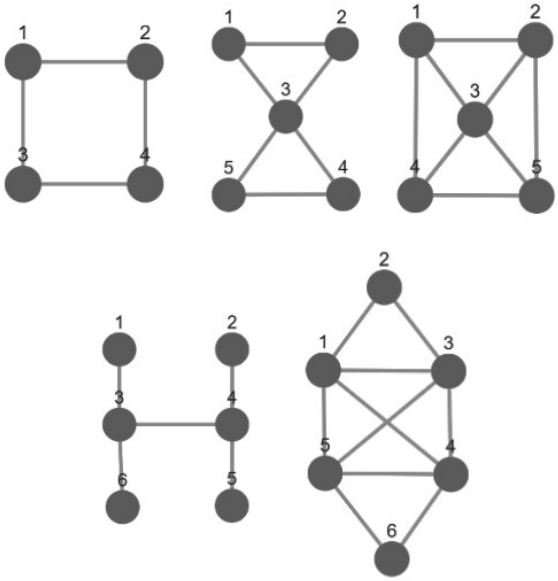

Figure 3.17: List of minimal asymmetric graphs without two vertices with an automorphism group of size 8

3.3.5 Graphs with an automorphism group of size 12

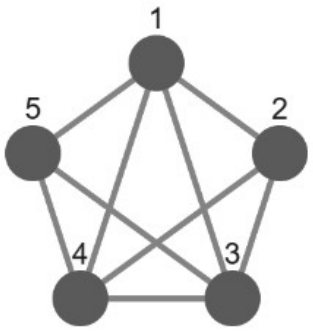

Figure 3.18: Minimal asymmetric graph without two vertices with an automorphism group of size 12

3.3.6 Graphs with an automorphism group of size 16

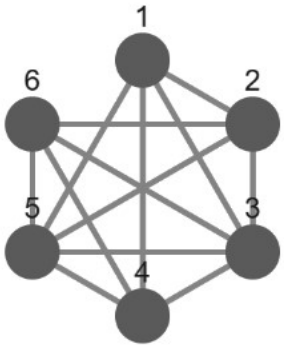

Figure 3.19: Minimal asymmetric graph without two vertices with an automorphism group of size 16

3.4 Minimal asymmetric graphs without three vertices

There are nineteen connected graphs, that can be obtained from minimal asymmetric graphs, when we delete three vertices from all of them. They have an automorphism group size of 2, 4, 6, 8, 12, or 24.

3.4.1 Graphs with an automorphism group of size 2

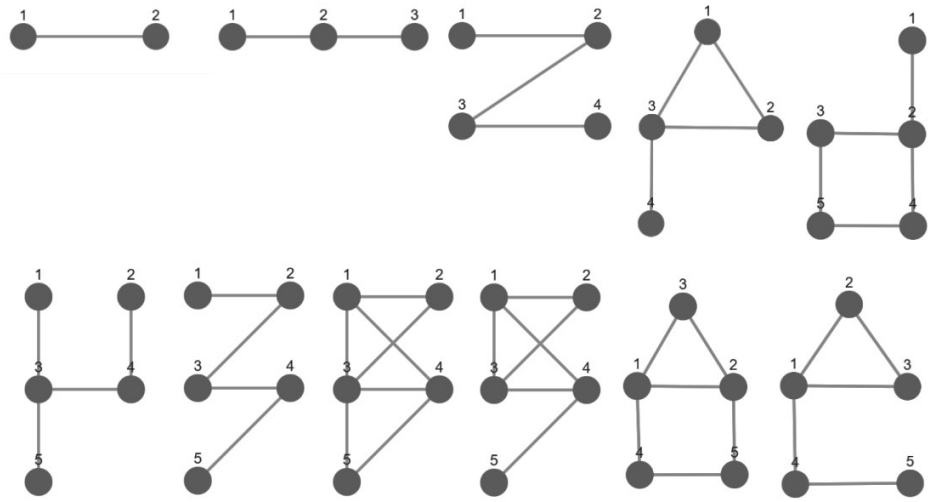

Figure 3.20: List of minimal asymmetric graphs without three vertices with an automorphism group of size 2

3.4.2 Graphs with an automorphism group of size 4

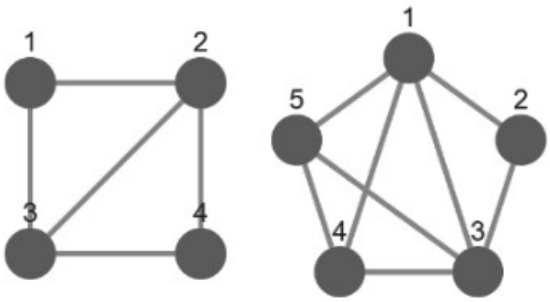

Figure 3.21: List of minimal asymmetric graphs without three vertices with an automorphism group of size 4

3.4.3 Graphs with an automorphism group of size 6

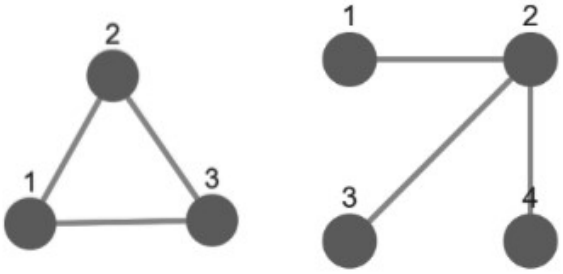

Figure 3.22: List of minimal asymmetric graphs without three vertices with an automorphism group of size 6

3.4.4 Graphs with an automorphism group of size 8

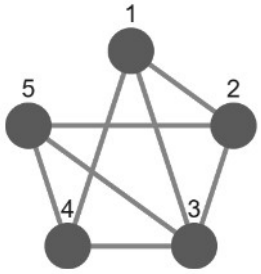

Figure 3.23: Minimal asymmetric graph without three vertices with an automorphism group of size 8

3.4.5 Graphs with an automorphism group of size 12

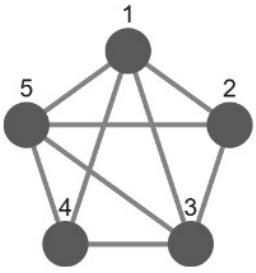

Figure 3.24: Minimal asymmetric graph without three vertices with an automorphism group of size 12

3.4.6 Graphs with an automorphism group of size 24

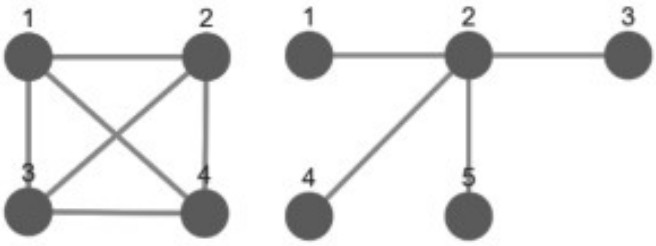

Figure 3.25: List of minimal asymmetric graphs without three vertices with an automorphism group of size 24

3.5 Minimal asymmetric graphs without four vertices

There are eight connected graphs, that we can obtain from minimal asymmetric graphs, when we delete four vertices from all of them. They have an automorphism group size of 2, 4, 6, or 24.

3.5.1 Graphs with an automorphism group of size 2

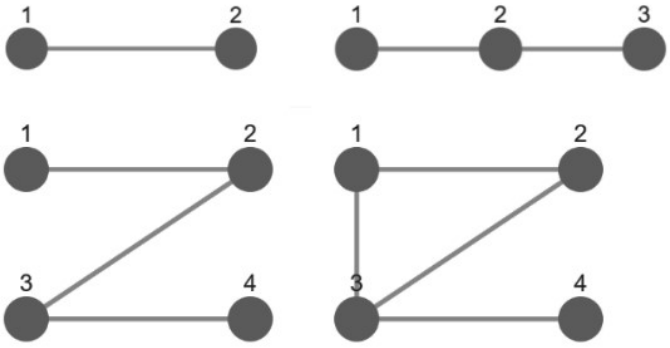

Figure 3.26: List of minimal asymmetric graphs without four vertices with an automorphism group of size 2

3.5.2 Graphs with an automorphism group of size 4

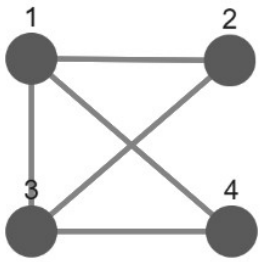

Figure 3.27: Minimal asymmetric graph without four vertices with an automorphism group of size 4

3.5.3 Graphs with an automorphism group of size 6

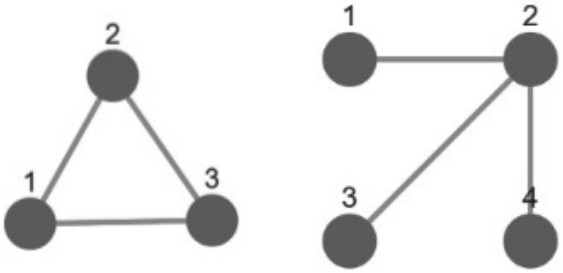

Figure 3.28: List of minimal asymmetric graphs without four vertices with an automorphism group of size 6

3.5.4 Graphs with an automorphism group of size 24

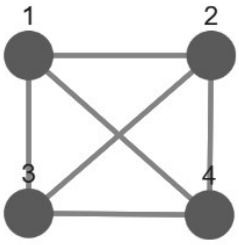

Figure 3.29: Minimal asymmetric graph without four vertices with an automorphism group of size 24

3.6 Minimal asymmetric graphs without five vertices

There are three connected graphs, that can be obtained from minimal asymmetric graphs, when we delete five vertices from all of them. Two of these graphs has an automorphism group of size 2 and one has an automorphism group of size 6.

3.6.1 Graphs with an automorphism group of size 2

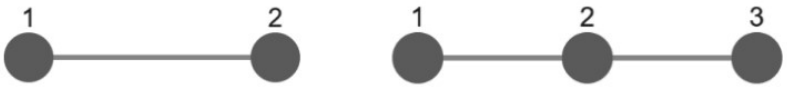

Figure 3.30: List of minimal asymmetric graphs without five vertices with an automorphism group of size 2

3.6.2 Graphs with an automorphism group of size 6

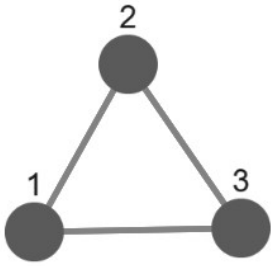

Figure 3.31: Minimal asymmetric graph without five vertices with an automorphism group of size 6

3.7 Minimal asymmetric graphs without six vertices

There is only one connected graph, that can be obtained from minimal asymmetric graphs, when we delete six vertices from all of them. That is caused by the number of vertices, that have minimal asymmetric graphs originally. This graph has an automorphism group of size 2.

3.7.1 Graphs with an automorphism group of size 2

Figure 3.32: Minimal asymmetric graph without six vertices with an automorphism group of size 2

Deleting more vertices, based on the text above, would lead to a graph with 1 vertex.