Effortless Learning of Fundamental Mechanics on Bicycle Derailleur

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Abstract. The derailleur is a well-known and essential component of bicycles, allowing riders to shift gears and adjust their pedaling resistance in different terrain and slope conditions. This article provides a very simple explanation of this mechanism. We focus on the mechanics of force and energy transmission. We intuitively introduce and use three notions of classical mechanics: circular motion, energy conservation law, and the relation between force and energy. We quantitatively describe the derailleur performance optimization function by analyzing its construction and energy conservation law in the considered system. Throughout the article, we provide visual aids and examples to illustrate the concepts and make them more accessible to readers/students with a limited technical background or absolute beginners in Newtonian mechanics. By the end of the article, readers will have a clear qualitative and quantitative description of the mechanics of the bicycle derailleur with a minimalistic effort to understand it. *This article is also suitable as a motivational lecture to high-school students to study physics at the University. It also allows practical illustration by using a bicycle in the lecture room.*

Introduction

Applying laws of mechanics to study peculiarities of the bicycle movement is certainly not a new idea [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. However, we aim to convey a piece of this knowledge in the simplest possible way. Meanwhile, Ref. [1, 2] have to introduce and explain such notion as torque, we will need just a much more intuitively accessible energy conservation law. Starting with this law, we show how and where the force and the torque are transferred as its consequence. In this way, even absolute beginners or high school students can easily quantitatively understand mechanic systems simplifying their daily lives.

Gear shifting is an essential part of cycling. After a very short time, one can recognize that when the chain is on the largest front gear and smallest rear gear, it boosts the distance covered with every push of the pedal (usually on the flat ground), even though it requires a greater force applied to the pedal. Contrarily, placing the chain on the smallest front gear and largest rear gear enables a slower (due to the shorter traveled distance per one pedal push) but steadier climb on steep hills with less pedal force.

Thanks to the human (evolutionarily highly successful) ability to create and use sophisticated tools [11], we automate the act of shifting gears to such an extent that we may dwell on it again when we get on a bicycle after a long time. Actually, when we learn how to use the derailleur, we do not need to solve physics equations in our head, we grasp the concept of the derailleur intuitively. Therefore, we can ask ourselves, how it really works and we can answer this question in a more detailed and quantitative way. This question is even more appealing if we realize that knowing how to use a tool and understanding the tool are two different things [12]. This material can be therefore viewed as an exercise applying fundamental laws of mechanics to the mechanical system that is well-known to almost everybody. In this way, the students can learn how the laws of physics explain known phenomena, which we commonly use to our advantage. On the other hand, such a simple mechanical device as the bicycle derailleur serves as a tool demonstrating the consequences of simple mechanical laws. Either way, pedagogically, this material can be viewed as an explanation using a suitable example, similar to e.g. Ref. [10, 13].

The material is organized as follows: Sec. 1 briefly introduces circular motion (mainly for self-containing reasons). Sec. 2 applies this knowledge to derailleur construction. Sec. 3 analyzes and exploits energy conservation. Sec. 4 concludes and relates results to empirical experience. Sec. 5 presents a simple DIY example with specific numbers.

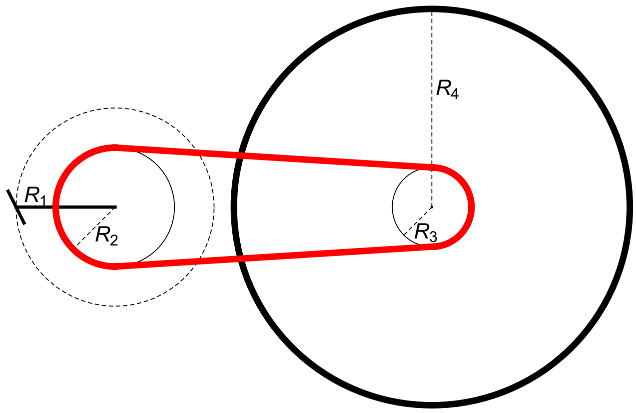

Figure 1. The basic geometry of the derailleur construction

We can realize that there are four wheels and also the four most important circular motions for the derailleur function. The first (perhaps somewhat hidden at first glance) is the motion described by the pedal. The dashed circle with a radius of R_1 in Fig. 1 represents this circle. The other two circles correspond to the front (with a radius of R_2) and rear (radius R_3) gears in Fig. 1. The chain, connecting the front and rear gear is plotted in red. Last but not least, the most recognizable, rear wheel with a radius of R_4, is also shown in Fig. 1.

The simple construction geometry of the bicycle derailleur also allows us to avoid more advanced mathematical techniques throughout explaining physics, which can (and will) be used to our pedagogical advantage.

1. Method: Circural motion

Observing the scheme of the mechanical construction in Fig. 1, it is clear that clarifying the mechanics of the derailleur requires recalling [13]/explaining the description of the circular motion. For this purpose, using the image in Fig. 2, we can realize that the distance s traveled on the circumference of a circle with radius R is related to the corresponding angle Θ (in radians) by the equation $s = R\Theta$.

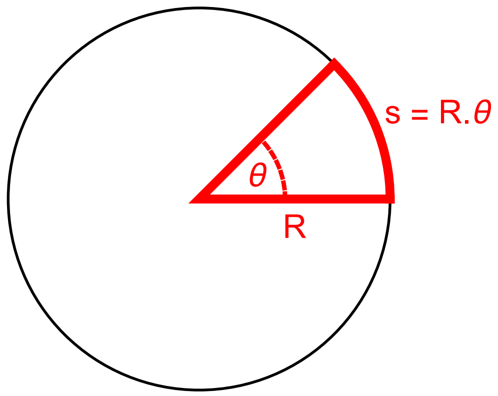

Figure 2. The relationship between the angle θ and its corresponding arc length s of a circle.

2. Method: Construction of a bicycle derailleur

Once we have clarified the circular motion, it is no problem to apply this concept to all four circular motions presented in Fig. 1, hence:

$$s_1 = R_1\Theta_1, \quad s_2 = R_2\Theta_2,$$
$$s_3 = R_3\Theta_3, \quad s_4 = R_4\Theta_4. \tag{1}$$

Of course, the construction of the derailleur mechanism allows for a series of simplifications. The (rigid) metal construction of the pedal ensures that $\Theta_1 = \Theta_2$, and the construction of the gearing on the rear wheel ensures that $\Theta_3 = \Theta_4$. The constantly taut chain connecting the front and rear gearing ensures that: $s_2 = s_3$. Putting it all together, we get:

$$s_4 = \frac{R_4}{R_1}\frac{R_2}{R_3}s_1. \tag{2}$$

This equation reveals the relationship between the length s_1 that we travel on the pedal and the length s_4 that travels the rear wheel in contact with the ground. If we further consider that R_1 and R_4 are constant (they usually change in the garage, but are quite difficult to change while cycling), we see that the change in the ratio between the radii of the front and rear sprockets R_2/R_3 controls how much of the length s_1 will eventually be manifested in the length s_4 transferred to the ground by the wheel[1]. Next, if we realize that the velocity is just the traveled distance over some time t, ($v = s/t$), and simply divide Eq. (2) by t, we see that:

$$v_4 = \frac{R_4}{R_1}\frac{R_2}{R_3}v_1, \tag{3}$$

1 Due to the nature of the matter, we are interested in the movement in which the movement of the rear wheel occurs only due to the movement of the pedal. The situation where the rear wheel rotates spontaneously represents an uninteresting case for our considerations, although we admit that it is a very pleasant case in practice.

and the same relationship provided for distances holds true also for the velocities.

3. Method: Transmission of force through conservation of energy

Of course, we need force to move with the bicycle from a stationary position. For our considerations, as well as in cases when we want to use the derailleur in real life, we will need a force F_1 always applied perpendicular to the rod (R_1) that connects the fully engaged pedal with the axis of rotation of the front gear (R_2). This force is associated with an "arrow"[2] with length F_1 in Fig. 3. It is also transmitted through the pedal to the gearing and acts on the tense chain with some force F_2. Through the chain, the force F_2 is further transformed to the force F_3 acting on the rear gear. Through the construction of the rear gear and wheel, the force F_3 is transmitted to the force F_4, which is transmitted directly on the road and allows us to accelerate[3].

The mentioned forces are, of course, somehow related. This relationship can be seen in several ways, and we focus on probably the simplest one using the conservation law of energy applied to the pedal. The key consideration for us[4] will be the fact that all the energy transferred from our foot to the pedal is transmitted through the mechanical construction of the bicycle and used for the actual movement of us and the bicycle. We do not consider the deformation of the construction and we neglect losses due to friction in the bearings[5].

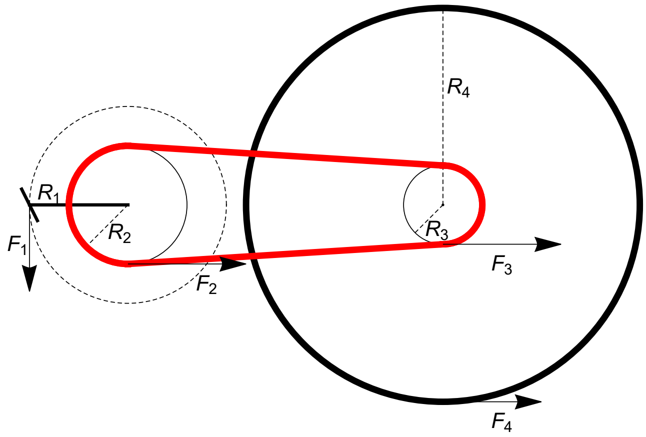

Figure 3. The transmission of force F_1 applied to the pedal through the forces transmitted by the chain F_2 and F_3, to the force F_4 acting on the road surface.

Next, let us use the fact, that the mechanical energy (or work) E is related to the applied force F over a distance s through the equation[6] $E = F.s$ [13], where we assume, of course, that a constant force F is applied in the direction of motion over a distance s. This relationship can be grasped intuitively, by walking a few flights of stairs without and then with a load, or by doing a few squads in the gravitational field of our planet Earth. However, the mechanical energy that is associated with the force transfer at each step (from one part of the construction to another), can be written as:

$$E_1 = F_1 s_1, \quad E_2 = F_2 s_2,$$
$$E_3 = F_3 s_3, \quad E_4 = F_4 s_4. \tag{4}$$

Since we assume a lossless energy transmission from the pedal to the rear wheel, it must be true that all energies are equal, so: $E_1 = E_2 = E_3 = E_4$. First, we focus on the

2 In the case of high-school students, who are not familiar with vector algebra, we can use this point to motivate the definition of the vector, or simply continue to work with an "arrow".
3 Again, from the nature of the discussion, it is clear that we are interested in situations when the wheel fully engages and does not slip.
4 As well as for the lifespan of our bicycle.
5 That is the reason to oil them properly.
6 Notice, once again, that the geometry of the bicycle allows us to use the simplest possible expression of work [13].

conservation of energy between the first and last step, $E_1 = E_4$, and we get:

$$F_1 s_1 = F_4 s_4. \tag{5}$$

Now, let us imagine a case where F_4 and s_4 are constant, which means that we are analyzing situations where the rear wheel of the bicycle exerts the same force F_4 on the road and always travels the same distance s_4. Eq. (5) tells us that its constant right-hand side can be achieved either by pushing a longer distance s_1 on the pedal with a smaller force F_1 or by pushing a shorter distance s_1 on the pedal with a greater force F_1. This is exactly what we observe and experience once we change gears.

Now, let us look at the derailleur function from a different perspective. If we combine our energy considerations from Eq. (5) with the ones previously formulated through the equations in Eq. (1), we get:

$$F_1 R_1 = F_2 R_2, \quad F_2 = F_3, \quad F_3 R_3 = F_4 R_4. \tag{6}$$

Among other things, the lesson learned from the first and the third equation consists of the equality of the torque magnitudes [13] between the pedal and the front gear resp. between the rear gear and the wheel. We can also realize that the chain equalizes the forces from the front gear F_2 to the rear gear F_3. In any case, from the equations expressed by Eq. (6), we obtain the relation between applied (F_1) and resulting (F_4) forces:

$$F_4 = \frac{R_1}{R_4}\frac{R_3}{R_2} F_1. \tag{7}$$

4. Conclusion, Interpretation and Discussion

To finally interpret and clarify the mechanism of the derailleur, let's rewrite equations (2) and (7) again:

$$s_4 = \frac{R_4}{R_1}\frac{R_2}{R_3} s_1, \quad F_4 = \frac{R_1}{R_4}\frac{R_3}{R_2} F_1,$$

so we can easily analyze them. Similarly to previous considerations, we assume constant values of R_1, R_4, s_4, and F_4. If that is the case, for a smaller ratio of R_2/R_3, we have to go a greater distance s_1 but apply a smaller force F_1, because the opposite ratio R_3/R_2 enters the second equation. This setting and behavior correspond to the choice of a "lighter" gear, where we have the chain on a smaller gear in the front and a larger one in the back. On the other hand, for a larger ratio of R_2/R_3, we need to go a smaller distance s_1, but we use a greater force F_1. Such a setting and behavior correspond to the so-called "heavier" gear. We can also see that since it is the ratio R_2/R_3 that matters, we can have different setups of gears (R_2 and R_3) manifesting very similar traveled distance s_4 and transmitted force F_4. This property will be notifiable in the example presented in the next section.

Notice that for our considerations, we focused only on the moving parts of the construction, completely ignoring details (such as moments of inertia [13] taking into account weight and mass distribution of wheels, etc.) or the solid-state structure of the frame, pedals, gears, or rear wheel, to which the mechanical parts transmitting force and energy are attached. We also omitted the part of the construction that keeps the chain tense [2] during gear changes, involving changing the radiuses R_2 and R_3. Although we do not take this into consideration, it is worth mentioning, that the property of metal[7] to maintain a constant shape and mass distribution[8] under normal conditions is crucial for the repeated transfer of force and energy. Although these ideas can be important for a detailed discussion of the efficiency and smoothness

7 Similar to other solid materials.

8 Due to the underlying atomic structure [14].

of the derailleur itself (and we encourage the reader in their analysis after reading our manuscript), they go way beyond our no-less-interesting and more elementary calculations and estimations.

5. Exercise: Specific numbers

To get a better sense of the exact numbers, let us look at a specific example[9]. If we want to find out how the transmitted force changes when we change the gear ratio, we only need to know the radii $R_{1,2,3,4}$. Measuring R_1 and R_4 is pretty straightforward, but measuring radii R_2 and R_3 can be more difficult. However, we can easily measure the number of teeth on the front and rear sprockets. Let us modify Eq. (7) in the following way:

$$F_4 = \frac{R_1}{R_4}\frac{2\pi R_3}{2\pi R_2}F_1 = \frac{R_1}{R_4}\frac{dn_3}{dn_2}F_1 = \frac{R_1}{R_4}\frac{n_3}{n_2}F_1, \quad (8)$$

where, in the second equality, we used the fact that the circumference of a gear is equal to the number of teeth $n_{2,3}$ multiplied by their length d. Of course, d is the same for both sprockets, because they are both connected by the same chain. Using the same trick in Eq. (4) leads to:

$$s_4 = \frac{R_4}{R_1}\frac{n_2}{n_3}s_1. \quad (9)$$

We found that to determine how the transmitted force (distance) changes, we can use the number of teeth on the front and rear gears for all gears.

The table Tab. 1 contains the specific number of teeth on all gears measured on the CTM Quadra 3.0 bicycle. On this occasion, it is also good to realize that counting teeth is an ingenious way to avoid imprecise measurements, but the physics formulated in the previous chapter would apply even if the front and rear gear were connected, say, by a belt[10].

Gear	1	2	3	4	5	6	7	8
n_2	22	31	44					
n_3	32	28	24	21	18	16	14	12

Table 1. Numbers of teeth on the front and rear gear wheels.

If we also measure the remaining two radii ($R_1 = 0.19\,m$ and $R_4 = 0.35\,m$), we can plot graphs showing the dependence of the transmitted force F_4 and the traveled distance s_4 on the gear ratio. Now, if we push the pedal through the imaginary semicircle path[11] $s_1 = 0.5\,m$, we see that the wheel always travels a greater distance than $0.5\,m$ in Fig. 4, regardless of the gear ratio. The slightest difference between s_1 and s_4 corresponds to the combination of front gear: 1, rear gear: 1, and the most significant difference to the combination of the front gear: 3 and rear gear: 8 as expected. Traveled distance for the later setup $s_4 = 3.38\,m$ corresponds nicely with our empirical experience.

To better understand specific, force-related numbers, let us examine the equation (8). In our case, let us assume $F_1 = 80\,N$, which corresponds to the force we need to exert to hold approximately an $8\,kg$ shopping bag[12]. If we push the pedal with a force of $F_1 = 80\,N$, we can see in Fig. 5 that the magnitude of the force transmitted to the rear wheel F_4 changes depending on the gear ratio according

9 This part can also be recommended to students as a simple do-it-yourself exercise.

10Of course, still considering no slipping on the gearwheel.

11(Being approximately the distance in which our foot fully engages with the pedal.

12Naturally, the force F_1 changes while cycling. For example, if a person with a weight of 80 kg pushes on the pedal with their entire weight, they would ideally be able to exert a force of about $800\,N$, which is an order of magnitude higher than our assumed $80\,N$.

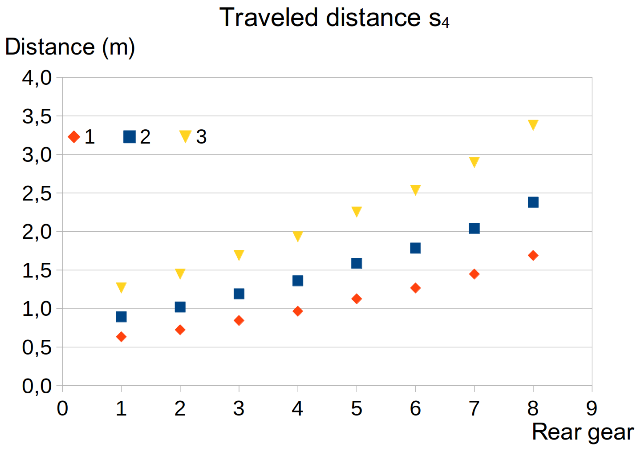

Figure 4. Traveled distance by the bicycle s_4 according to Eq. (9), using $R_1 = 0.19\,m$, $R_4 = 0.35\,m$, numbers of teeth n_2 and n_3 from the Tab. 1 and assuming $s_1 = 0.5m$. Red diamonds mark the setups with the first front gear, the second with blue squares, and the third with dark yellow triangles.

to Eq. (8).

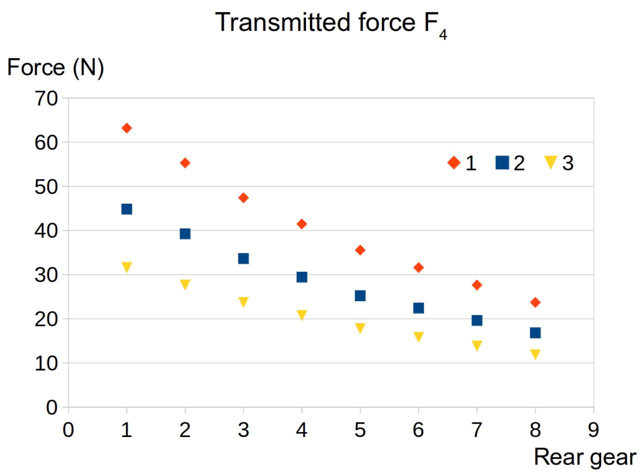

Figure 5. Transmitted force on the back wheel tire F_4 assuming $F_1 = 80\,N$, using numbers of teeth from Tab. 1 and Eq. (8). Labeling different colors corresponding to different front gears is adopted from Fig. 4.

Notice, that the combination of front gear: 1, rear gear: 1 leads to the greatest resulting force F_4 and the combination of front gear: 3, rear gear: 8 to the lowest transmitted force F_4, as empirically expected. This is why the combination of the front gear: 1 and the rear gear: 1 helps us to climb a steep hill. Notice also, that due to our specific derailleur construction, the resulting force F_4 is less than F_1 for any combination of gears. Fig. 4 and Fig. 5 also show a few cases of very similar traveled distance s_4 and resulting force F_4 for different combinations of gears, since it is the ratio of $R_2/R_3 = n_2/n_3$ that matters, as concluded at the very end of the Sec. 4.

In conclusion, thanks to elementary mechanics, we thoroughly analyzed the function of the bicycle derailleur. We recognized the difference between the roles of the chain transmitting force and the gear rack transmitting torque. Next, taking into account specific numbers, we were able to confirm and quantitatively enlighten the mechanics of the bicycle derailleur. Thanks to its simple construction, our analysis required the easiest mathematical formalism accessible to any (even a high school) student. By the end of this article, we would like to encourage students to study further the various aspects of bicycle mechanics described in Ref. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13].

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Brief description of the author's work

František Herman, born in 1989, earned his PhD from the Comenius Uni. In Bratislava, Slovakia, in 2017. He is doing research in the Cond. matter physics, theory, mainly in the field of superconductivity. His passion is also teaching. The material in this manuscript was developed together with his students as a motivational lecture for high school students.

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