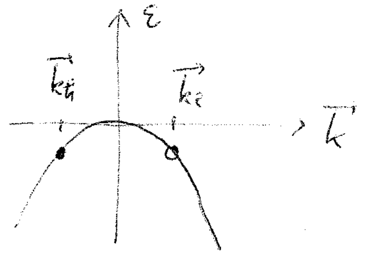
8 POLOVODIČE

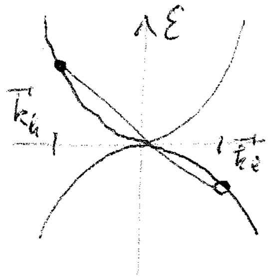
A) Pojem diery

v blízkosti maxima valenčného pásu je užitočný pojem diery
= absencia elektrónu



absencia elektrónu s vlnovým $\vec k_e$
= excitácia s vlnovým (diera) $\boxed{\vec k_h = -\vec k_e}$

energia diery:



$$\boxed{\varepsilon_h(\vec k_h) = -\varepsilon_e(\vec k_e)}$$

- rýchlosť diery $\vec v_h = \frac{1}{\hbar}\frac{\partial \varepsilon_h}{\partial \vec k_h} = -\frac{1}{\hbar}\frac{\partial \varepsilon_e(\vec k_e)}{\partial \vec k_h} = -\frac{1}{\hbar}\frac{\partial \varepsilon_e}{\partial \vec k_e}\frac{\partial \vec k_e}{\partial \vec k_h} = \frac{1}{\hbar}\frac{\partial \varepsilon_e}{\partial \vec k_e} = \vec v_e$
- hmotnosť diery $(m_h^*)_{ij} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon_h(k_h)}{\partial k_i^h \partial k_j^h} = -\frac{1}{\hbar^2}\frac{\partial^2 \varepsilon_e(k_e)}{\partial k_i^e \partial k_j^e} = -(m_e^*)_{ij}$

(teda v blízkosti maxima pásu je $m_h^* > 0$)

- pohybová rovnica $\hbar\dot{\vec k}_e = -e(\vec E + \vec v_e \times \vec B)$

$$\downarrow$$

$$\boxed{\hbar\dot{\vec k}_h = +e(\vec E + \vec v_h \times \vec B)}. \quad \longleftrightarrow \text{ častica s nábojom } +e$$

- prúd nesený elektrónom $= -e\vec v_e$

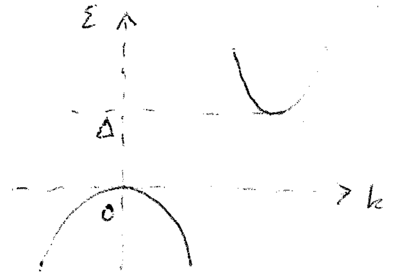
  dierou $= +e\vec v_h$

$\downarrow$

- celkový prúd $\vec j = -e \underbrace{\int_{\text{obs. stavy}} \frac{d^3k}{4\pi^3}\vec v_k}_{\text{elektrónový obraz}} = +e \underbrace{\int_{\text{neobs. stavy}} \frac{d^3k}{4\pi^3}\vec v_k}_{\text{dierový obraz}}$

(oba výrazy ekvivalentné, lebo $\int_{BZ} \frac{d^3k}{4\pi^3}\vec v_k = 0$)

B) Hustota stavov (obidva spiny) vo vodivostnom pásme Si

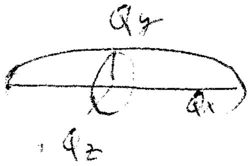


$$\varepsilon(\vec{q}) = \Delta + \frac{\hbar^2 q_x^2}{2m_L} + \frac{\hbar^2 (q_y^2 + q_z^2)}{2m_T}$$

$\vec{q}$ = odchýlka od minima

Počet stavov s energiou $< \varepsilon + \Delta$:

$$G_e(\Delta+\varepsilon) = \frac{2}{8\pi^3} \cdot \frac{4\pi}{3} \cdot q_x q_y q_z \cdot 6$$



$$q_x = \sqrt{\frac{2m_L\varepsilon}{\hbar^2}}, \quad q_y = q_z = \sqrt{\frac{2m_T\varepsilon}{\hbar^2}}$$

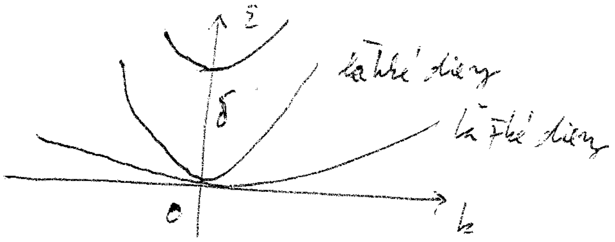
6 ekvivalentných miním

$$\longrightarrow \quad G_e(\Delta+\varepsilon) = \frac{2}{\pi^2} \frac{\sqrt{2m_L}}{\hbar} \frac{2m_T}{\hbar^2} \varepsilon^{3/2} \rightarrow N_e(\varepsilon) = \frac{dG_e}{d\varepsilon}$$

$$\boxed{N_e(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon-\Delta}}$$ kde $\boxed{m_e = (36\, m_L m_T^2)^{1/3} \approx 1{,}08m}$

C) Hustota stavov (obidva spiny) – valenčné pásmo Si

dierový jazyk:



Pre energie $< \delta$:

$$\varepsilon_h(\vec{k}) = Ak^2 \pm \sqrt{B^2k^4 + C^2(k_x^2k_y^2 + k_y^2k_z^2 + k_z^2k_x^2)}$$

$$\boxed{N_h(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon}}$$ kde $\boxed{m_h \approx 0{,}59m}$

1) Koncentrácia vlastných nosičov

elektróny: $$n = \int_\Delta^\infty d\varepsilon\, N_e(\varepsilon)\, f(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}\int_\Delta^\infty \frac{d\varepsilon\,(\varepsilon-\Delta)^{1/2}}{e^{\frac{\varepsilon-\mu}{T}}+1}$$

Predpoklad: $\boxed{\Delta-\mu \gg T}$ $\longrightarrow$ $$n = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}\int_\Delta^\infty d\varepsilon\,(\varepsilon-\Delta)^{1/2} e^{\frac{\mu-\varepsilon}{T}}$$

$$\int_0^\infty dx\,\sqrt{x}\,e^{-x} = \Gamma\left(\tfrac{3}{2}\right) = \frac{\sqrt{\pi}}{2} \longrightarrow \boxed{n = 2\left(\frac{m_e T}{2\pi\hbar^2}\right)^{3/2} e^{\frac{\mu-\Delta}{T}}} \quad (1)$$

diery: $$p = \int_{-\infty}^0 d\varepsilon\, N_h(-\varepsilon)\,[1-f(\varepsilon)] = e^{-\frac{\mu}{T}}\int_0^\infty d\varepsilon\, N_h(\varepsilon)\, e^{-\varepsilon/T}$$

predpoklad: $\boxed{\mu \gg T}$

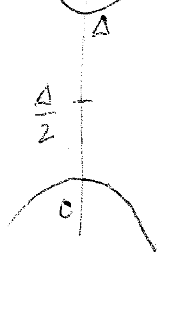
$$\longrightarrow \boxed{p = 2\left(\frac{m_h T}{2\pi\hbar^2}\right)^{3/2} e^{-\mu/T}} \quad (2)$$

z (1,2) máme $$np = 4\left(\frac{T}{2\pi\hbar^2}\right)^3 (m_e m_h)^{3/2} e^{-\Delta/T}$$

Ale $n = p = n_i \longrightarrow$ $$\boxed{n_i = 2\left(\frac{T}{2\pi\hbar^2}\right)^{3/2}(m_e m_h)^{3/4} e^{-\frac{\Delta}{2T}}} \quad (3)$$

Porovnaním 1,3 máme: $$m_e^{3/2} e^{\frac{\mu-\Delta}{T}} = (m_e m_h)^{3/4} e^{-\frac{\Delta}{2T}}$$

$$\boxed{\mu_i = \frac{\Delta}{2} + \frac{3T}{4}\ln\frac{m_h}{m_e}} \longrightarrow$$ úvahy platia pre $\Delta \gg 2T$



Vzorce z tejto strany možno písať aj takto:

$n_0(T) = 2\left(\frac{m_e T}{2\pi\hbar^2}\right)^{3/2}$ $\qquad n = n_0 e^{\frac{\mu-\Delta}{T}}$

$p_0(T) = 2\left(\frac{m_h T}{2\pi\hbar^2}\right)^{3/2}$ $\qquad p = p_0 e^{-\mu/T}$

$\mu_i = \frac{\Delta}{2} + \frac{T}{2}\ln\frac{p_0}{n_0}$ $\qquad n_i^2 = n_0 p_0 e^{-\frac{\Delta}{T}}$

E) Prímesné stavy v Si

fosfor $P = [Ne]\,3s^2 3p^3$ ... o 1 elektrón a 1 protón viac
hliník $Al = [Ne]\,3s^2 3p^1$ ... o 1 elektrón a 1 protón menej
} ako Si

• Fosfor: efektívny hamiltonián (Donor)

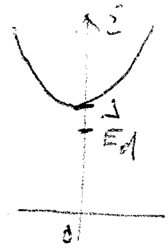
$$\boxed{H_{eff} = \varepsilon_c\left(\frac{\vec{p}}{\hbar}\right) - \frac{e^2}{4\pi\varepsilon_0\varepsilon_R r}} \quad (*)$$

↳ potenciál od nekompenzovaného protónu

Δ vodivostný pás

aproximujeme jediným minimom s izotrópnou hmotnosťou m^*,

$$\rightarrow \quad E_d = \Delta - \frac{m_e^* e^4}{2(4\pi\varepsilon_0\hbar)^2\varepsilon_R^2}$$ ... energia tohto stavu / donorovej hladiny



$\varepsilon_R \approx 12$, $m_e^* \sim 0{,}2$ ⟶ takmer 1000x menšia väzbová energia ako v atóme H

$$\boxed{\Delta - E_d \approx 44\,\text{meV}}$$ pre P v Si ⟶ $\Delta - E_d \ll \Delta$

$\Delta - E_d \sim T$ pre izbové teploty

Priblíženie (*) funguje dobre, lebo rozmer tohto stavu

$\sim \frac{m}{m^*}\varepsilon_R \cdot$ (Bohrov polomer) $\sim 100\,Å \gg$ mriežková konštanta

• Hliník: efektívny hamiltonián (Akceptor)

$$H_{eff} = \varepsilon_v\left(\frac{\vec{p}}{\hbar}\right) + \frac{e^2}{4\pi\varepsilon_0\varepsilon_R r} \xrightarrow{\text{dierový obraz}} \boxed{H_{eff} = \frac{\vec{p}^{\,2}}{2m_h^*} - \frac{e^2}{4\pi\varepsilon_0\varepsilon_R r}}$$

valenčný pás

odpudivý potenciál (chýbajúci protón)

tohto stavu diery

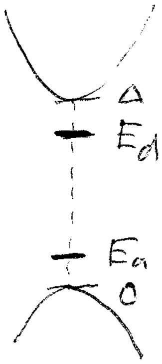
$$E_a = +\frac{m_h^* e^4}{2(4\pi\varepsilon_0\hbar)^2\varepsilon_R^2}$$

elektrónová energia

v základnom stave chýbajúci elektrón obsadí stav s energiou E_a

Al v Si: $E_a = 57\,\text{meV}$

:) Štatistika nosičov náboja v dopovanom polovodiči



E_d – viazané donorové stavy

E_a – viazané akceptorové stavy

elektrónový obrat

N_d = koncentrácia donorov

N_a = koncentrácia akceptorov

n_d = konc. obsadených viazaných donorových stavov

p_a = konc. dier vo viazaných akceptor. stavoch

$\boxed{n_d = ?}$

viazané donorové stavy môžu byť obsadené iba jedným elektrónom (pre 2 elektróny veľké Coulombovské odpudzovanie)

priemer. počet obs. stavov v 1 donore $= \dfrac{n_d}{N_d} = \sum_j P_j N_j$ $\qquad P_j = \dfrac{1}{Z} e^{-\frac{E_j - \mu N_j}{T}}$

$$Z = \sum_j e^{-\frac{E_j - \mu N_j}{T}}$$

j	0	↑	↓	↑↓
N_j	0	1	1	2
E_j	0	E_d	E_d	∞

$$\frac{n_d}{N_d} = \frac{2e^{-\frac{E_d-\mu}{T}}}{1+2e^{-\frac{E_d-\mu}{T}}} = \boxed{\frac{1}{\frac{1}{2}e^{\frac{E_d-\mu}{T}}+1} = \frac{n_d}{N_d}} \qquad (4)$$

$\boxed{p_a = ?}$ $\qquad \underbrace{p_a + n_a}_{} = 2N_a \longrightarrow \dfrac{p_a}{N_a} = 2 - \dfrac{n_a}{N_a}$

n_a → obsadené akceptorové viazané stavy

j	↑↓	↑	↓	0
N_j	2	1	1	0
E_j	$2E_a$	E_a	E_a	∞ ← Coul. odpudzovanie dier

$$\frac{n_a}{N_a} = \frac{2e^{-\frac{2E_a-2\mu}{T}} + 2e^{-\frac{E_a-\mu}{T}}}{e^{-\frac{2E_a-2\mu}{T}} + 2e^{-\frac{E_a-\mu}{T}}}$$

$$\rightarrow \boxed{\frac{p_a}{N_a} = \frac{1}{\frac{1}{2}e^{\frac{\mu-E_a}{T}}+1}} \qquad (5)$$

Elektroneutralita:

$$(6)\quad \boxed{\underbrace{p + p_a + N_d}_{\text{kladný náboj}} = \underbrace{n + n_d + N_a}_{\text{záporný náboj}}} \quad \text{... rovnica pre } \mu$$

Predpokladáme $\boxed{\begin{array}{c} \Delta - \mu \gg T \\ \mu - 0 \gg T \end{array}}$ → potom:

$$\boxed{\begin{array}{l} n = n_0(T)\, e^{\frac{\mu-\Delta}{T}} \quad (7) \\ p = p_0(T)\, e^{-\mu/T} \quad (8) \end{array}}$$

Rovnice 4, 5, 6, 7, 8: 5 rovníc pre 5 neznámych n, p, n_d, p_a, μ

Po dosadení do (6):

$$\boxed{p_0(T)\, e^{-\frac{\mu}{T}} + \frac{N_d}{2e^{\frac{\mu-E_d}{T}}+1} = n_0(T)\, e^{\frac{\mu-\Delta}{T}} + \frac{N_a}{2e^{\frac{E_a-\mu}{T}}+1}} \qquad (9)$$

Pre konkrétnosť uvažujme $N_d \gg N_a$ (polovodič typu n)

a) nízke teploty: vidíme, že $\mu \sim E_d$ → $p \approx 0$

$$\frac{N_a}{2e^{\frac{E_a-\mu}{T}}+1} \approx N_a$$

$$\boxed{\frac{N_d}{2e^{\frac{\mu-E_d}{T}}+1} = n_0\, e^{\frac{\mu-\Delta}{T}} + N_a}$$

Zavedieme $z = e^{\frac{\mu-E_d}{T}}$ → možno písať:

$$2z^2 + z\left(1 + \frac{2N_a}{n_0} e^{\frac{\Delta-E_d}{T}}\right) - \frac{N_d-N_a}{n_0} e^{\frac{\Delta-E_d}{T}} = 0$$

riešenie

$$z = \frac{-\left(1 + \frac{2N_a}{n_0} e^{\frac{\Delta-E_d}{T}}\right) + \sqrt{\left(1 + \frac{2N_a}{n_0} e^{\frac{\Delta-E_d}{T}}\right)^2 + 8\frac{N_d-N_a}{n_0} e^{\frac{\Delta-E_d}{T}}}}{4}$$

Tvar $z = \dfrac{-a + \sqrt{a^2+b^2}}{4}$

Ultranízke teploty, $a \gg b \longrightarrow z = \dfrac{N_d}{2N_a}$

$$\boxed{\begin{aligned} \mu &= E_d + T \ln \frac{N_d}{2N_a} \\ n &= \frac{n_0 N_d}{2N_a}\, e^{-\frac{\Delta - E_d}{T}} \end{aligned}}$$ ... oblasť A

- **Nízke teploty**, $a \ll b \longrightarrow z = \sqrt{\dfrac{N_d}{2n_0}}\, e^{\frac{\Delta - E_d}{2T}}$

$$\boxed{\begin{aligned} \mu &= \frac{\Delta + E_d}{2} + \frac{T}{2} \ln \frac{N_d}{2n_0} \\ n &= \sqrt{\frac{n_0 N_d}{2}}\, e^{-\frac{\Delta - E_d}{2T}} \end{aligned}}$$ ... oblasť B

b) **vysoké teploty**: očakávame $E_d - \mu \gg T$

$\mu - E_a \gg T$

Potom (3) sa redukuje na tvar

odkiaľ $\boxed{\begin{aligned} n - p &= N_d - N_a \\ np &= n_i^2 \end{aligned}}$ $\qquad \left(\begin{aligned} n &= n_i e^{\frac{\mu - \mu_i}{T}} \\ p &= n_i e^{\frac{\mu_i - \mu}{T}} \end{aligned} \right)$

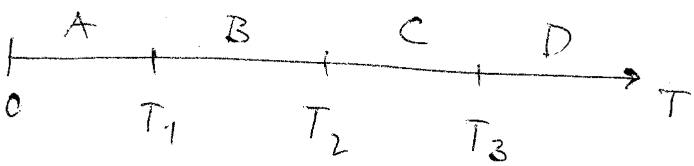
Preto
$$\boxed{\begin{aligned} n &= \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \\ p &= \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \end{aligned}}$$

$$\boxed{\operatorname{sh}\left(\frac{\mu - \mu_i}{T}\right) = \frac{N_d - N_a}{2n_i}}$$

Uvažujme stále $N_d \gg N_a$.

- **Vysoké teploty**: $N_d \gg n_i \longrightarrow \boxed{n = N_d, \quad p = \dfrac{n_i^2}{N_d}}$ ... oblasť C
- **Ultravysoké teploty** $N_d \ll n_i \longrightarrow \boxed{n = p = n_i}$ ... oblasť D

Sumár



$$0 < T \ll T_1: \quad n = \frac{n_c N_d}{2N_a} e^{-\frac{\Delta - E_d}{T}} \longrightarrow \mu \approx E_d, \quad n < N_a$$

$$T_1 \ll T \ll T_2: \quad n = \sqrt{\frac{n_c N_d}{2}}\, e^{-\frac{\Delta - E_d}{2T}} \longrightarrow \mu \approx \frac{E_d + \Delta}{2}, \quad N_a < n < N_d$$

$$T_2 \ll T \ll T_3: \quad n = N_d \longrightarrow \text{plne ionizované donory}$$

$$T_3 \ll T: \quad n = \sqrt{n_c p_0}\, e^{-\Delta/2T} \longrightarrow \text{"intrinsický" polovodič}$$

odhad pre T_1, T_2 T_3 dostaneme z porovnania vzorcov pre n v hraničiacich oblastiach, napr.

$$N_d = \sqrt{n_c(T_3)\, p_0(T_3)}\, e^{-\Delta/2T_3} \quad \text{atď.}$$

Tak máme:

$$T_1 = \frac{\Delta - E_d}{\ln\left(\frac{2N_a^2}{n_c N_d}\right)}$$

$$T_2 = \frac{\Delta - E_d}{\ln\left[\frac{n_c(T_2)}{2N_d}\right]}$$

$$T_3 = \frac{\Delta}{\ln\left[\frac{n_c(T_3)\, p_0(T_3)}{N_d^2}\right]}$$