Chapter 42
Chladni Plate and Chladni Patterns—A Research Review of Theory, Modelling, Simulation and Engineering Applications

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1 Introduction

Cymatics is the study of visual patterns which are generated by sound or vibrations waves. Very unique patterns are created at different frequencies of sound and vibration of systems in use. The scientist named Ernst Chladni studied the sound patterns of different plates. The plates included shapes like circular, quadratic and rectangular plates. The experiments were performed by fixing plates with fingers at different points, which resulted in creating nodal lines at these points. These experimental results were published in the year 1787 in Entdeckungen uber die Theorie des Klanges with 11 plates and a total of 166 figures [1].

Chladni also developed new musical instruments and used it to demonstrate Chladni patterns. After conducting number of experiments his first musical instrument was developed in the year 1790 and was named Euphon. After that he developed another musical instrument in 1799 and named as the Clavicylinder [1].

Chladni also worked on vibration of bars and plates, and it became his main field of interest. After working in this area of plate vibrations his discovery, the Chladni

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plate, is still associated with his name. Before Chladni, the person who observed these vibration figures, was Leonardo da Vinci. This has been endorsed in his notebook, and Galilei treated them in a longer passage of his Dialogo sopra I due massimi sistemi. But it can be said that first time the most systematic research of vibration patterns was carried by Chladni [2].

This Chladni experiment is used for the visualizing of acoustical stationary waves on a plate. The Chladni plate is a typical case of two-dimensional wave systems. There are various applications of this experiment. It includes different fields of study like music, nano-mechanics, micro-particle manipulation and vibration analysis. With advancement in the design of vibration generators, nowadays the Chladni plate experiments are performed with the help of mechanical or electromagnetic vibrators.

In many industrial applications the vibration and its control is a major concern. The study and analysis of vibration phenomenon requires costly set of equipment which includes accelerometers and Data Acquisition system. The typical experimental setup and software for post-processing for Chladni plate pattern generation is shown in Fig. 1.

The experimental set-up includes a mechanical vibrator for generating vibrations coming from, a signal generator. A plate is fixed at the centre point and connected to the vibrator as shown in Fig. 1.

This paper reviews the research work related to Chladni plates. The phenomenon demonstrated by Chladni plate, and the patterns generated by vibrations are part of Cymatics. So, the research on Chladni plate and Cymatics are closely associated. Many areas related to Chladni plate research are overlapping with Cymatics research and its corresponding applications. Considering the above constraints, the research related to Cymatics and Chladni plates can mainly classified in following major application areas.

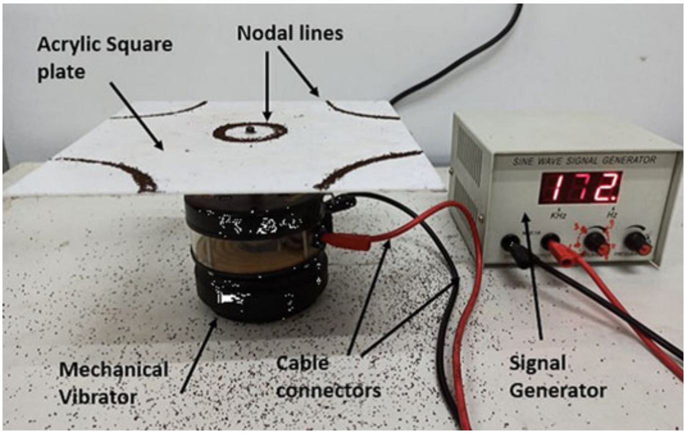

Fig. 1 Typical Chladni plate set-up for experimentation [3]

These areas are (i) musical field and musical instruments, (ii) micromanipulation, (iii) human body healing techniques, (iv) modal analysis of plates, (v) visualization of sound and vibrations, (vi) damage detection, (vii) condition monitoring and fault diagnostics, (viii) vision-based vibrometry, (ix) development of non-contact measurement techniques for noise and vibrations, (x) Chladni patterns and machine learning techniques, (xi) theoretical and mathematical methods related to Chladni plates, (xii) inverse Chladni effect, (xiii) Cymatics, Chladni patterns and study of spiritual practices like meditation, mantra chanting, binaural beats, etc., (xiv) visual arts using Chladni patterns, (xv) fluid vibrations, (xvi) quantum mechanics, (xvii) sound and light patterns, (xviii) image recognition techniques for Chladni patterns, (xix) ultrasonics and infrasonic applications, (xx) Lissajous figures and analysis, (xxi) vibration of plates and membranes, (xxii) speech recognition and audio analysis, (xxiii) image processing.

This review paper covers, some of the major areas. These areas are musical fields and musical instruments, Chladni plate, patterns and their applications. Another area covered are mathematical models and simulations.

2 Mathematical Equations of Chladni Plates

In 1932, Robert C. Colwell worked with square plate and found out that, if the vibrations are provided to an electric circuit of quartz crystals, due to the oscillations, the crystals split into different parts. To observe these nodal lines, the lycopodium powder or sand is used. He further formulated a general solution for square Chladni plates as shown in the Eq. (1),

$$A \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{a} + B \cos \frac{n\pi x}{a} \cos \frac{m\pi y}{a} = 0, \tag{1}$$

where a is side of square, m and n are whole numbers and A and B are arbitrary numbers. If $A = B$, then symmetric forms can be generated [3].

In 1933, Colwell formulated the symmetry of square plate along its diagonal by putting. If m is even and n is odd or vice versa, then one of the two diagonals of square will be nodal line. Further, he experimentally formed patterns along single and both diagonal and median lines by changing m and n. He was the first one who experimentally formed these intricate patterns [4]. In 1934, Colwell with Bryant used magneto-strictive oscillations on Chladni plate which the help of which they were able to generate complex patterns on square as well as on circular plate [5]. In 1937 with Hill and Colwell created a circuit to vibrate small quartz plate using electrical field and similarly these plates were vibrated using mechanical impact and they have found that the patterns obtained with both of the methods were exactly similar [6].

In 1938, Stewart and Colwell generated patterns on a circular plate or circular membrane and compared them with the patterns formed using graphical method [7]. As the below equation,

$$W = AJ_n(kr)\cos n(\theta - \alpha_n) - bJ_m\left(k'r\right)\cos m(\theta - \alpha_m) = 0, \tag{2}$$

(where A and B are numbers which explains the intricate nature of nodal lines on circular plate, θ varied from 0° to 90°, m and n are diameters of circle) gives fairly accurate results. The square plate's Chladni patterns have their equations for each and every pattern but is unable to generate every single case. But, by modifying the equation it gives approximate patterns but are much closer to physically formed pattern. The modification consists of rewriting the equation with

$$y = Jm\left(k'r\right), \quad y = KJ_n(kr), \tag{3}$$

where $K = \frac{A\cos\ n(\theta-\alpha_n)}{B\cos\ m(\theta-\alpha_m)}$.

In 1954, the study done by Warburton explained about all possible cases which can be applied to a square shaped Chladni plate. In total, 15 cases where studied which are formed by considering fixed supported, freely supported and free. For numerical analysis, Rayleigh method was utilized, where assumption considered was, waveforms are similar to beams [8].

In 1955, Jensen used loudspeaker for the generation of Chladni patterns on the Chladni plate. He used a loudspeaker of 15 W with audio-oscillator and power-amplifier and placed Chladni plate on the top of the speaker with central point fixed. The shapes considered where square, circular, triangular and hexagonal [9].

No figure generated is an identical match to the mathematically generated figure is due to the boundary conditions assigned to the plate. In 1956, Waller used solid carbon sublimation method to generate free vibrations and studied the patterns generated with this method [10]. If the crystals of snow were considered as vibrating system, then crystals show similar phenomena as Chladni plate. At nodes, the concentration of growth is present, and the symmetric nature of the crystal is due to the nature of vibrations. This was studied by Smalley [11]. In addition to this Smalley has mentioned that the figures of nodes with 12 symmetrical arrangements can be generated by Waller with a Chladni plate of hexagonal shape [11].

In 1979, Langley and Taylor studied the Chladni figures formation, when the random vibrations are provided to the square plate. The analysis is conducted using both theoretical analyses, using reverberation field method and experimental analysis, where the random vibrations are provided to the plate at centre point [12]. The finite element study of plate was studied by Sansalone et al., where they have considered impact-echo technique for the study of transient wave propagation. The calculation of displacement formed by stress wave was calculated using finite element method and green's function and favourable agreement was observed in this study [13]. In 1993, Bardell studied the Chladni patterns formulated on parallelogram shaped plate. In this study, total 108 cases were considered, 6 modes of plate, at 3 aspect ratios', at 6 skew angles. For the numerical analysis, Ritz method was used, and the method gives quick convergence. In addition to this, other frequency dependent parameters like skew angle, Poisson's ratio and aspect ratio were also studied [14]. Other than this, decent amount of work is done on mode shapes, natural frequency of plate and other structures like cantilever beam where damage present in the structure is detected with the help of eigen parameters [15] and damage detected by change in the curvature of mode shapes [16].

3 Mathematical Modelling and Numerical Simulations

Ernst Chladni started his work with circular plate under free vibrational conditions. In 1969, Leissa written a case file on vibration of plate, which contains the information about circular plates, elliptical plates, rectangular plate, parallelogram plate, triangular plates, etc. [17]. The focus of majority of the articles were axisymmetric vibrations. The first exact solution for non-axisymmetric vibrations of a circular plate with a single nodal diameter was given by Kovalenko. The thickness of plate used, Kovalenko was linearly varying.

Elishakoff modelled a mathematical expression of axisymmetric vibrations of clamped plates. He is the first one who formulated closed loop solution [18].

Later, in 2004, Storch et al. formulated solution of non-axisymmetric condition for circular plate under clamped edge, simply supported edge. The equation governing the forced vibration of a circular plate,

$$\begin{aligned} & D\nabla^4 w + 2\frac{\mathrm{d}D}{\mathrm{d}r}\frac{\partial}{\partial r}\nabla^2 w + \nabla^2 D \nabla^2 w \\ & - (1-v)\left[\frac{1}{r^2}\frac{\mathrm{d}^2 D}{\mathrm{d}r^2}\frac{\partial^2 w}{\partial^2 \theta} + \frac{1}{r}\left(\frac{\mathrm{d}^2 D}{\mathrm{d}^2 r}\right)\frac{\partial w}{\partial r} + \frac{1}{r}\frac{\mathrm{d}D}{\mathrm{d}r}\frac{\partial^2 w}{\partial r^2}\right] \\ & + \delta\frac{\partial^2 w}{\partial t^2} = q(r,\theta), \end{aligned} \tag{4}$$

where $q\ (r, \theta)$ is loading, $D(r)$ flexural rigidity, polar radius r. If the radius is R, the mode shape W must satisfy the boundary conditions, $w = \frac{\mathrm{d}w}{\mathrm{d}r} = 0$, at $r = R$. The solution for clamped condition is,

$$w(r,\theta) = \frac{qR^4}{192D}\left(\frac{r}{R}\right)\left(1 - \frac{r^2}{R^2}\right)^2 \cos\theta \tag{5}$$

For simply supported edge, the boundary conditions are $W = M_r = \mathrm{O}$ at $r = R$, where M_r is bending moment,

$$W(r,\theta) = \frac{q_1 R^4}{192D(3+v)}\left(\frac{r}{R}\right)\left(1 - \frac{r^2}{R^2}\right)\left[7 + v - (3+v)\left(\frac{r}{R}\right)^2\right]\cos\theta, \tag{6}$$

where v is Poisson's ratio [19].

Papkov [20] formulated vibrations of rectangular orthotropic plate with free edges. He studied Kirchhoff love approximation. By considering a problem and solved natural frequencies and mode shapes of rectangular orthotropic plate on the basis of asymptotic law [20].

Arango and Reyes [21] studied the stochastic models for Chladni figures. He used the stochastic differential equations model which considers the wiggly motion of particles on plate, when resonated at natural frequencies. In this paper, stochastic model of Chladni's experiment is defined by the SDE as,

$$\mathrm{d}X_t = \mu_\phi(X_t)\nabla\phi(X_t)\mathrm{d}t + \sigma_\phi(X_t)\mathrm{d}W_t, \tag{7}$$

where $X_0 = x$, where ϕ is vibrational mode, W_t is standard two-dimensional Brownian motion. He also studied the similarity between movement of particles in experiments and Brownian motion [21].

Using Daubechies wavelet elements, Nastos et al. [22], studied the propagation of wave of laminated composite plates. He used finite wavelet domain (FWD) method used to generate mathematical model of laminated composite plates. In addition to this, the discretization of equation of motion is formulated for 1st order shear deformation laminated plate theory. Also, advantages of FWD method also discussed [22].

Nguyen et al. [23] studied the Kirchhoff love plate model considering three boundary conditions clamped, simply supported and free. For spatial discretization, Nguyen utilized second order centred finite difference scheme. The algorithm used was explicit-predictor corrector and implicit Newmark Beta time step. Afterwards, the Numerical methods were compared with experimental results [23].

Using derivations of Leissa, Pereira et al., conducted simulations on thin plate for energy finite element method (EFEM), energy spectral element method (ESEM) and modal superposition method (MSM). At lower frequencies, approximations of energy do not give attractive solutions compared to MSM; but at higher frequencies, results obtained by energy methods are preferred [24].

4 Musical Fields and Musical Instruments

One of the oldest and very interesting area of work is related to music. The various instruments used in music including, violin, tabla, piano and guitar are studied including their sound producing characteristics. Chladni patterns helps to understand the performance of these musical instruments at different amplitudes and frequencies.

The work on violin is done by Molin et al., in which the effect of violin's material and geometrical properties are studied with respect to vibration modes. Chladni patterns are generated and compared with FEA simulations. Here modal analysis of wooden blanks which are used for making violin is done using Chladni plate techniques [25]. Figure 2 shows the arrangement of collecting the image dataset using tonoscope with sound as an input.

One such work done by Gough includes the study of the violin and the Chladni patterns generated by it. Here the elements considered are bowed string, the

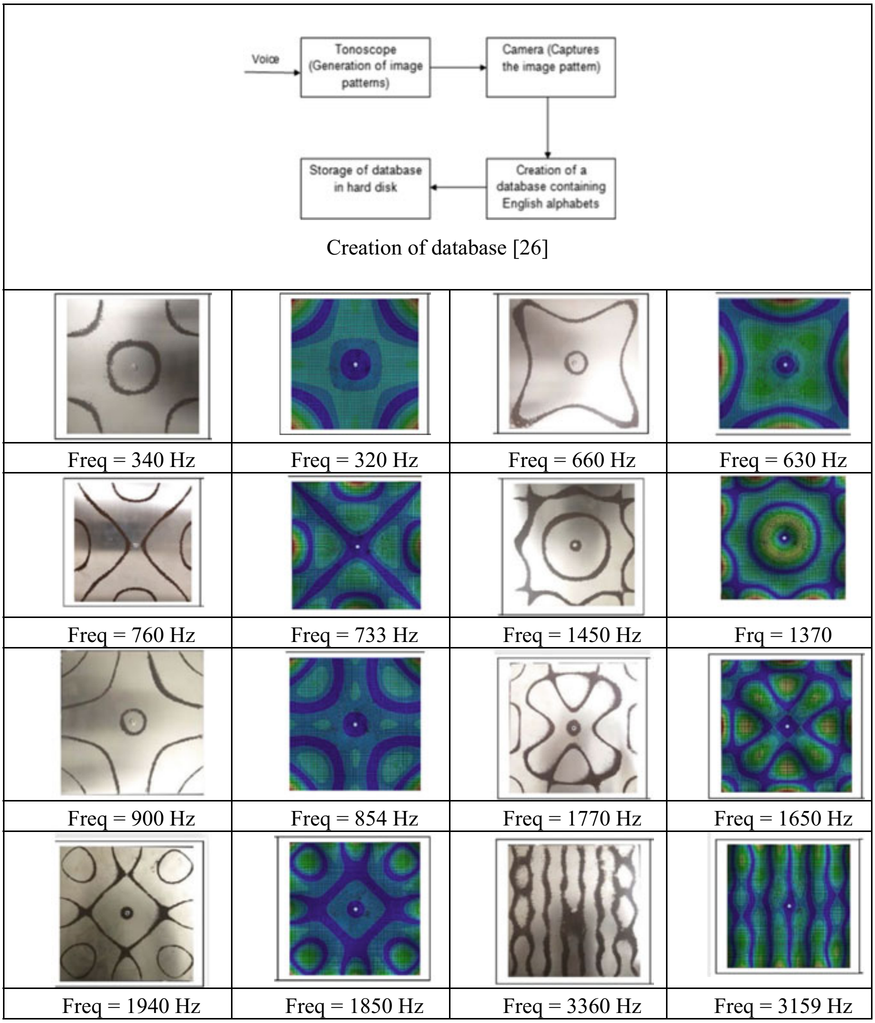

Fig. 2 Chladni patterns obtained at different frequencies and simulation results [37]

supporting bridge, the hollow shell and acoustics modes generated while playing the violin. Chladni patterns are generated and compared with nodal patterns simulation done using Finite Element Analysis (FEA). The results showed that the inherent damping property of the violin is important in characterizing the normal modes of violin and based on which weak and strong coupling limits can be separated [26].

Another instrument on which Chladni patterns are studied is a guitar, the work of Okuda and Ono analyses the guitar's mode shapes. Here the guitar's bracing, which internally supports the soundboard, is studied. The different patterns of guitar's bracing are studied by forming Chladni patterns on the soundboard. Here the guitar's top boards are made with composites of polyurethane foam and carbon fibres. Replicas of two wooden guitars, one with bracing and another without bracings are analysed for their mode shapes. The experimental results are also compared with FEA. The results show that composite top boards can be used for guitar replicating the properties of wood [27].

Wadaiko is the name of the traditional Japanese drum. The study is carried by Ono et al. on Wadaiko vibration characteristics. In this work the comparison of sound characteristics of artificial wood plastic, Keyaki and Sen wood is carried out. The Chladni experiment is done on wood plastic. The results obtained were sufficient to show that wood plastic can be a good replacement for traditional wood for making the Wadaiko which is traditional Japanese drum [28].

Musical instruments and its study in relations to the sound generated is also another major application of Cymatics. Ioan Curtu et al. investigated classical guitar bodies and its tonal qualities. The Chladni patterns are obtained for the acoustic boxes using sand on the surface. The work demonstrated the impact of the strutting system on the mode shapes as well as on dynamic response [29].

Another work done by Anna Danihelova et al. uses Chladni patterns for modal analysis of wooden plates which are used in different musical instruments. Here the patterns are compared with holographic interferometry technique [30].

Another instrument that undergone study is drumhead. Worland R. has conducted experimentation on drumhead by placing them on the top of stands and placed Styrofoam in between the stand and drumhead. The patterns were formed with the help of 3-in speaker on 11-in and 22-in drumhead [31].

In the study done by Munoth et al., the setup was prepared with the help of speaker which is placed inside the container. The membrane is present on the top surface of the container. As the membrane vibrates and patterns are generated when the sound is produced, and these generated patterns were captured for further studies. In this experiment, sound data of English alphabets are given, and the patterns generated by that data were studied using mean square distance histograms. The patterns show unique features except V and W [32].

The research work done by Perrin et al. includes the modal analysis of a 20.7 cm diameter steel gamelan gong. For modal analysis, four different methods are used. These methods are electronic speckle-pattern interferometry (ESPI), Finite Element Method (FEM), Laser Doppler Vibrometry (LDV) and Chladni patterns. The results show that the scanning LDV method can be a used for investigating lower order modes. Also, all four methods shown agreeable results for different modes [33].

5 Chladni Plates, Patterns and Their Applications

By seeing the published work of Chladni, whole lot of researchers studied this phenomenon of Chladni patterns. Various tools are used in the process, including FEA, numerical simulation, signal processing, modal analysis, etc. The conventional methods for vibration analysis and operational modal analysis are available. As per

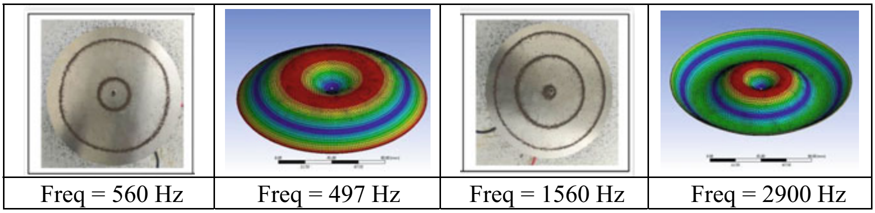

Fig. 3 Chladni patterns on circular plate experimental and simulation results [37]

the work done by Pasca et al. the operational modal analysis can be done with the help of Python based software Py_OMA for evaluation modal parameters [34]. Another method can use of intelligent algorithms for automatic modal parameter estimation, this work is done by Rosso et al. [35]. On similar line deep learning techniques can be used for vibration analysis applications. Rosso et al. comprehensively reviewed these deep learning methods used particularly for structural health monitoring [36].

This section reviews such work carried out on Chladni plates, the generated patterns on these plates and applications of these methods in various fields.

The Chladni figures can be used to visualize vibrational mode shapes. The work done by Atul Malviya and Kiran Wani includes the modal analysis of square plate and circular plate. The simulation results are compared with Chladni experiments. Figures 2 and 3 shows the Chladni patterns formed experimentally on the aluminium square plate and aluminium circular plate, respectively.

The simulation is carried out with the help of ANSYS software. The comparison of experimental and simulated mode shapes for square plate are as shown in Fig. 2. Figure 3 shows the experimental as well as simulation mode shapes of circular plate which are closely matching with the experimental mode shapes shown.

In Fig. 3, the simulation of circular plate is shown, these shapes are closely matching with the experimental results. The results show that the Chladni patterns are increasingly complex towards higher frequencies. The simulated results and Chladni experiment patterns are in close agreements for square plates as well as circular plates [37]. These simulation results are compared with the experimental Chladni results, and this comparison is shown in Table 1. The %age error is below 10%.

Another work done by Sato et al., not only used the concept of Chladni plate to analyse vibrations of paperboard, but also used for deriving the value of in-plane shear modulus and dynamic Poisson's ratio. This vibration method is more suitable for non-uniform and imperfect sheet materials such as paperboard. This research work shows that the Chladni figures also contains the information on elastic properties of material and the Poisson's ratio can be determined with the help of curvature of the nodal line of Chladni figures [38]. The work done by Oh and Kim includes the study on Chladni patterns created on Chladni plate made up of paper and polystyrene board. The patterns were taken on range of 110–1200 Hz in difference of 10 Hz. Though figures obtained are not identical but have some features similar to the patterns generated with violin bow. The application areas of Chladni figures are, in

Table 1 Percentage error between experimental and simulation work for rectangular plate [37]

S. No.	Experimental frequency (Hz)	Simulation frequency (Hz)	% Error
1	340	320	5.88
2	660	630	4.54
3	760	733	3.55
4	900	854	4.60
5	1000	953	4.70
6	1450	1370	5.51
7	1770	1650	6.77
8	1940	1850	4.63
9	2540	2290	9.84
10	3250	2994	7.87

Cymatics-related therapy, where body is exposed to sound and magnetism and also in autism-related study [39].

Other than mechanical vibrator and violin bow and piezoelectric transducer, Chladni pattern can be excited by speaker. In the study conducted by Gurukiran et al., he attached a speaker to the centre of the Chladni plate compared generated patterns with numerically generated modal patterns. Maximum error obtained in this study is 11% [40]. A lot of researchers worked in the area of Chladni plate and its modal analysis. The work done by Ma et al. does the modal analysis of a circular plate using three different ways. These are Chladni patterns, FEA and experimental modal analysis. The results show that all of the frequencies of Chladni plate, as eigen parameters are dependent three methods produce similar results [41].

Adding the mass to the plate tends to change the frequencies of Chladni plate, as eigen parameters are dependent on material properties and mass of the plate and this effect is studied by Shubham and Wani. They have found out that, if the stiffeners are placed on the plate, at antinodes, then the patterns were generated at higher frequencies than that of the patterns generated at the plate without the stiffeners [42]. Increase in frequency tends to produce Chladni Patterns of complex nature, and at Ultrasonic range of vibrations, these patterns become so complex that, for comparing the experimental and simulations results, one has to use image processing. In recent study covered by Joshi and Wani include the formation of Chladni patterns at ultrasonic ranges where they have validated the experimental shapes formed on stainless steel plate in ANSYS Workbench with modal as well as with Harmonic response analysis [43].

In another work done by Anirudh dixit and Wani where they have formed the Chladni figures on circular rubber membrane and in addition to this, they have conducted acoustic simulations of the same in ANSYS Workbench in modal and harmonic analysis. Furthermore, they have used deep learning algorithms like ResNetV2, Xception, etc., for classification of experimentally generated Chladni patterns [44]. The work done by Kaczmareka et al. is related to the modal analysis of two different material plates. One of them is made up of medium-density fibreboard (MDF) which is an engineered wood product. Another plate is from circular saw blade. Different modes of vibrations are visualized using Chladni patterns. The results are compared, with the conclusion that the circular saws with large holes appeared with two resonant frequencies with same nodal numbers. Another conclusion drawn is that the square plate with centre point fixed have different Chladni patterns as compared to the square plate constrained at its edges [45].

In the study done by Lemeš et al. applied theory of vibrations on circular plate to the centrifugal impeller, to study mode shapes on the centrifugal impeller. Further author concluded that same study can be implemented other pumps, wheels of rail waggon, hard discs of computer, etc., for the classification and identification of modes of vibrations on the structures [46]. The work done by Tuan et al. compares the eigen modes and resonant mods of a square plate. Chladni figures are obtained with the derived wave function. The wave function is derived with the help of eigen mode expansion method. The derived wave function is also utilized for numerically constructing the wave patterns observed in microwave resonators [47].

In the study conducted by Tseng, the utilization of Chladni plate is achieved for the point interaction in quantum billiards. The overlapping of eigen function is studied, which contributes for the generation of new eigen function due to shift in eigen value, is also numerically validated. The good agreement between experimental nodal line patterns and numerical nodal line patterns is achieved [48]. Another approach to do the modal analysis of the circular plate is used by Trejo-Mandujano et. al. In this work, the model is combined using Huygens principles and ideas of the Bohr atom. The results are compared with the experimental results which are closely matching with each other [49]. The work done by Tuan et al. includes the formation of a modern Chladni figures and resonant frequency spectrum, which are rectified by theoretical model. The experimental results also conclude that modern Chladni figures avoids crossing of nodal lines whereas classical Chladni figures forms a regular grid [50]. One more research work carried by Huan et al. is based on obtaining resonant frequencies of thin plate by varying the impedance of the exciter. The Chladni patterns are reconstructed with the help of Helmholtz equation. Furthermore, the resonant frequencies derived from experiments and resonant wave numbers are combined to conclude the dispersion relationship for the flexural wave of the vibrating plate [51].

The work done by Skrodzki et al. is related to creating three dimensional Chladni figures. After the theoretical formulation to obtain the Chladni patterns in two dimensions, an image production technique is applied to convert 2D pattern into 3D Chladni patterns [52]. Another major application of Cymatics is manipulation or patterning of microparticles. Shi et al. studied utilization of standing surface acoustic waves for patterning cell and microparticles. This work has successfully demonstrated the new patterning technique called as acoustic tweezers used to arrange cells and microparticles in the desired patterns. This has lot of applications in biology, chemistry, engineering and material science. The work uses standing surface acoustic waves [53]. The micromanipulation can also be done by using combination of Chladni plate and machine learning technique like reinforcement learning. This method is investigated by Latifi et al. In this work a model free acoustic manipulation method is successfully used on Chladni plate for micromanipulation of particles [54]. If the Chladni plate is submerged in fluid, then radiation force of acoustics and lateral weight becomes commanding entities at sub-mm scale. This experimental study is covered by Latifi et al. where statistical model was created which links with particle location and vibrational frequencies for particle manipulation. With the use of this model, single as well as group of particles can be manipulated in fluid medium using Chladni plate phenomenon [55].

The use of neural networks for the prediction of motion of particles is done by Kopitca, where the patterns generated on square plate having side of 50 mm and thickness of 0.525 mm. Using this model, different shapes where formed which were of simple geometry and Latin letters [56]. In the experiments conducted by Podolak, the plate used to generate Chladni patterns was circular shaped plate having 24 cm in diameter and made up of non-magnetic coated steel plate. For the sprinkling particles at first salt was used and afterwards 4 g of iron, then 4%, 8% and 12% of nickel was adulterated in 4 g of iron by weight. A gauss meter was used to measure external magnetic fields. To move the particles from outer to inner rings, localized magnetic field was given externally. To convey the particles at higher frequencies, reduction in magnetic field is required, compared to lower frequencies [57].

When the vibrations are provided to the Chladni plate through an actuator, the particles will relocate themselves, when they are excited to the specific frequency. Using the principle, manipulation of particles can be done, and this work is done by Latifi et al. In their study, they have implemented methods such as open-loop control, closed loop control and linear programming methods. In open-loop control, pre-calculate model was utilized and for closed loop, visual feedback loop was considered. In addition to this for path tracing as well as for path following, other models like model predictive control were implemented [58]. Using Chladni principles manipulation of mixture of particles can be done in such a way that, particles separated from each other. The study done by Hou et al. utilizes inverse Chladni plates' principles for the multiple particle separation. In their study, they have considered mixture of different specimens like two kinds of silica sand, two heavy materials namely titanium alloy particles and polystyrene particles. Observations concluded that particles separate from the mixture. Heavy particles occupied the centre space while lighter particles move at the antinodes or vice versa based on the mixture considered [59].

To understand the Chladni patterns it is essential to study the vibration analysis of plates and membranes. Many researchers worked in this area and one of the researcher Reutskiy carried out the vibration analysis for arbitrarily shaped membranes. In this work a latest numerical technique for the analysis of non-homogeneous membranes is developed. Here amalgamation of finite difference method and conformal mapping is utilized as a solver for the boundary value problem [60]. Some of the researchers are also tried to solve the problem of analysis of plate under forced vibration conditions. The work carried out by D'Alessio uses a numerical procedure to solve this problem. The method consists of two step finite volume schemes with boundary condition and fixed centre of a square shaped aluminium plate. The results show closely matching with experimental results [61].

Contaminants present on the solar panel can be cleaned with the application of Cymatics and this study is done by Babu et al. In their study, the experimental study is done with the support of Finite Element Analysis. To remove the contaminants from the surface of solar panel, the panel is vibrated near to its natural frequency, because of this the effective cleaning was achieved [62]. Apart from conventional patterns which are observed on the plates of various shapes, inverse Chladni patterns are also obtained in rather unique conditions. One such work is carried out by van Gerner et al., in which inverse Chladni shapes are formed on resonating plates even in the absence of air currents. The concept is studied theoretically and experimentally [63]. The inverse Chladni patterns formed on the plate are due to the air currents generated by the vibrations, carrying along particles towards antinode points. This was first mathematically explained by Lord Rayleigh in 1884. But the study is done by van Gerner et al. consists of numerical explanation of air present around Chladni plate. The dominance created by Newtonian forces on heavy particles results in to Chladni patterns, but in case of inverse Chladni patterns, Stokesian forces are dominant [64].

The relation between Chladni patterns and node shapes is very close. The points where the sand particles are collected are the node points and nodal lines. These shapes clearly indicate the mode shapes of the plates. Modal analysis plays a vital role in the investigation in structural health monitoring as well as in damage detection. On the similar lines, Fan et al. have done a comprehensive review of the techniques for plates considering modal parameter-based damage identification methods [65].

A unique case for vibratory system is vibration analysis of a double panel structure. Lase Doppler Vibrometry technique has a disadvantage of shifting resonance frequencies to other frequencies under the influence of drastic changes in temperatures. To overcome this limitation, Rzepecki et al., method of Chladni figures for doing modal analysis of a double panel structure. Gray-Level Co-occurrence Matrix (GLCM) method is used to convert the experimental images into binary images. Finally, the results are compared with ANSYS simulation. The work concluded that Chladni method can be used for modal analysis, with the limitations of necessary conditions of planar and smooth surface and sensitivity to air flow [66].

For vibration analysis non-contact type of vibration measurement methods such as laser vibrometry can be used. Similarly, modal analysis can be done using various experimental methods. With the advancement in machine learning techniques, computer vision has lot of application in modal analysis and ultimately in structural analysis. Yang et al. carried out the full order dynamic model of the cable in the structures of bridges using computer vision techniques. For this work dynamic loads such as vehicle load over the bridge is considered as an input to analyse and predict the behaviour of the cable. A video processing approach and unsupervised machine learning to extract maximum modal parameters is used. Finally, the results are validated with experiments [67]. Laser Doppler Vibrometry (LDV) can be used for seismic investigation. One such work is done by Paul Sava et al. Conventional seismic instruments requires landed instrument such as seismometers and geophones whereas here the LDV technique which can sense the motion at a distance without contact with the ground is used for experimentation [68]. In another investigation of an oil pan of diesel engine having two cylinders, Duvigneau et al. worked on the mode's shapes. Using Laser Doppler Vibrometer, the mode shapes are generated at the free-free boundary condition, on a thin aluminium plate. In addition to this, numerical and analytical results were compared and found out that different modes are obtained due to the negative or positive superposition of modes [69].

The paper written by includes Luo et al. The method of optical level to investigate the modes shapes formed on the Chladni plate. For experimentation, they considered circular acrylic plate of three different diameters and are excited harmonically at the centre. The slope of the mode shape was derived with the length of spot created by the laser on the screen, as the laser emitted by the light was reflected with perpendicularly placed vibrating plate, which is placed away from the screen. Afterwards the by integrating the slope, mode shapes were determined [70].

In the area of structural health monitoring vibration analysis is used for damage detection. Rafalle et al. suggest a machine learning approach for structural health monitoring. This work has used machine learning and statistical technique to characterize the acceleration measurement directly in the time domain. Laboratory tests are carried out on beam structures and compared with simulated results [71]. The Chladni like patterns formed on the water surface are studied by Lei et al. and these patterns are known as unconventional Chladni patterns. For the study, the standing bulk acoustic wave was given to cylindrical glass tube which contains fluid which was water and are excited at few megahertz. The patterns achieved experimentally are similar to that of simulations in COMSOL [72].

Lin et al. provided vibrations to the container containing fluids. The Chladni like patterns knowns are faradays wave are formed on the free surface of fluids. The reaction performed was blue bottle reaction, which is redox in nature. At nodal regions, glucose was present and at antinodal regions, oxygen present in surrounding gets dissolved into the solution [73]. The paper written by McGowan et al. elaborate the tools which are used to visualize the sound, namely, VU Meter, Cymatics images, Chladni Plate, etc. VU meter gives the information related to the loudness of sound. In Cymatics the patterns generated on water or on any other materials were studied. Cymatics is interesting as the patterns obtained are intricate and attractive in nature and small fluctuations in inputs, results give noticeable change in patterns [74].

Tonoscope and Soledine are the software which are used to visualize the Chladni figures on square plate and patterns on water in circular shape, respectively. With the use of these software, Dubravko Miljković generated patterns from noise generated from Aircraft Engine. In this study, Dubravko Miljković used noise generated from Piston Engine, turboprop engine, Turbojet engine and Turbofan engine and analysed their waveforms, spectrums, spectrograms and Cymatics patterns. Further author mentioned that, if an engine is operating in a correct way, then the patterns produced are clear and distinct, but if not, then patterns are different and unappealing [75]. Further work in the area of damage detection is done by Wojtczak et al. on structural adhesive joints. Here it has been investigated that the effect of frequency of wave, on damage identification in adhesive joints of single lap. It is observed that, the value of chosen wave frequency has a strong influence on effectiveness of measurements [76]. One of the non-contact types of modal analysis techniques is video based evaluation.

This method can be used in combination with interferometry. Chen et al. used camera-based heterodyne interferometry and acoustic excitation for modal analysis. Acoustic waves are used for excitation of the structure for exciting cantilever beam made up of steel and modal parameters are controlled using heterodyne interferometry. This technique is compared with Finite Element Analysis and Laser Doppler Vibrometry for vibrational modal analysis [77].

Considering the huge database of images generated during the experimentation of the Chladni plate experimentation, it is obvious to analyse this image database. The best method to do this is implementation of various machine learning techniques. To select a proper method of machine learning is important to achieve the desired results. In the field of machine learning lot of new algorithms are developed and field of fault diagnostics is also using these techniques. Convolutional neural network (CNN) is being widely used for fault diagnostics. Chen et al. integrated convolutional neural network (CNN) with Extreme Learning machine (ELM). CNN is used to extract high level features whereas Extreme Learning machine (ELM) improves the classification performance applied for detecting the faults for datasets of gearbox and motor bearings [78]. Vision-based modal analysis is using different algorithms, one of them is multiple vibration distribution synthesis. Aoyama et al. used multiple vibration distribution synthesis methods for vibration analysis and tested on truss structure bridge model. This method is able to evaluate high order modal analysis [79].

In case of vibration analysis and fault diagnostics the data acquired may be without labels, in such cases unsupervised learning methods are used. Li et al. used this method for gear fault diagnostics. Here raw signal of vibration is directly used and augmented deep sparse encoder (ADSAE) is utilized for fault diagnosis. It is seen that this method increases the network generalization and accuracy [80]. Ultra-high-speed cameras are also used in some cases to carry out the modal analysis. Li et al. investigated the modal properties using ultra-high-speed cameras. In this work, the experiments are done on rotating mirrors for damage detection [81]. Image recognition is also used successfully for the fault diagnosis. Zhou and Cheng used image recognition for fault classification of bearings under variable loading conditions. A neural network classifier is used to perform the fault diagnosis and the method proved to be effective for the fault diagnosis [82]. Computer vision is becoming very useful in detecting the faults particularly with the help of slowing down the video. Davis et al. used small often imperceptible motion of any component which is recorded by a high-speed camera, to detect material properties. In this Visual Vibrometry approach, the modes of vibration are extracted from the video to infer the material properties [83]. One of the techniques used for extracting vibration data and also the sound of the vibrating object is by using the high-speed motion camera analysis. Using only high-speed video of the object minute vibrations as well as sound is recovered. Thus, the vibrating object is converted into visual microphone and quality of the sound recovered is evaluated using intelligibility and SNR metrics [84]. Many researchers have done the fault diagnostics using neural networks and also done fault classifications. By using Winger ville distribution, Wu and Chiang developed a magnificent system for the fault detection of IC Engines by utilizing feature extraction and probability neural network. In both time as well as in frequency domain analysis, the Wigner ville distribution provides high resolution of instantaneous energy density [85].

Another very interesting application of Chladni plate and its pattern is investigated by Igea and Cicirello. This application is related to plates made up of orthotropic material. There is direct impact of condition used during manufacturing of the orthotropic panels and properties of the panels and respective structure made using these panels. Here plates made from three materials plywood, aluminium and FR4 are investigated using Chladni pattern methods to predict their manufacturing variability, in terms of elastic constants. The results showed close approximation with experimental and simulated results [86]. The work done by Janusson et al. uses Chladni pattern concept to demonstrate the concept of quantum mechanical computer and its correspondence with atomic orbital shape. Here the Chladni plate and patterns are used as cross-sectional representation of atomic orbitals [87]. One more innovative application of Chladni plate available in the literature is investigating textile resonators. The work is done by Karsten Neuwerk et al. The main focus of this work is to design textile resonator for absorption of noise at lower frequencies with multiple absorbing mechanisms. Here the textile resonators are analysed using Chladni patterns and laser vibrometry. The results showed successful implementation of textile resonators as sound absorbers by changing their textile structures [88].

6 Conclusion

The literature reviewed can be summarized with the conclusion that Chladni plate analysis has major applications in the area of modal analysis, fault diagnostics, Cymatics, micromanipulation. Also, it is clear from the literature that number of methods, tools and techniques can be employed to obtain the mode shapes, including Finite Element Analysis (FEA), Numerical Techniques, Wave Equations, etc. The latest trends include the use of vibration analysis and measurements using non-contact type of measurement techniques such as Laser Doppler Vibrometry (LDV) and electronic speckle-pattern interferometry (ESPI). Finally, it can be concluded that Chladni plate and its pattern formation is being applied in all major areas of engineering and in future, it is going to be used along with modern techniques of image recognition and machine learning algorithms.

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