Computational analysis of efficient gated recurrent neural networks

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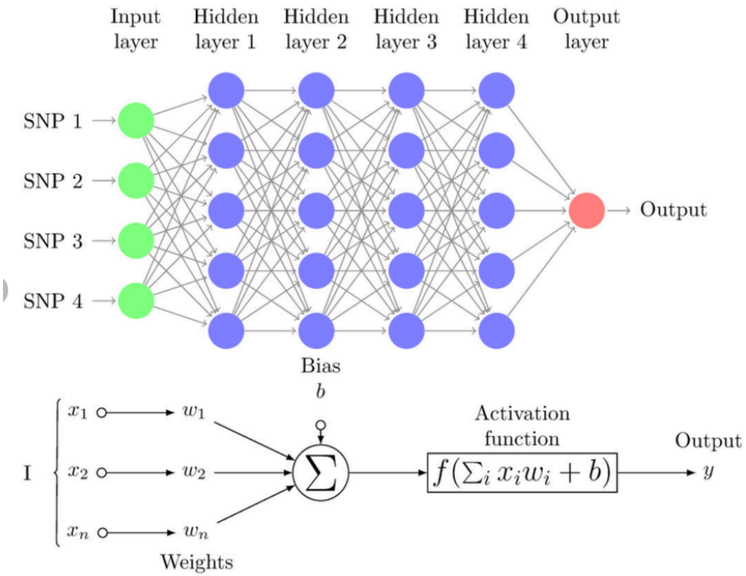
Multilayer perceptron



Recurent neural network

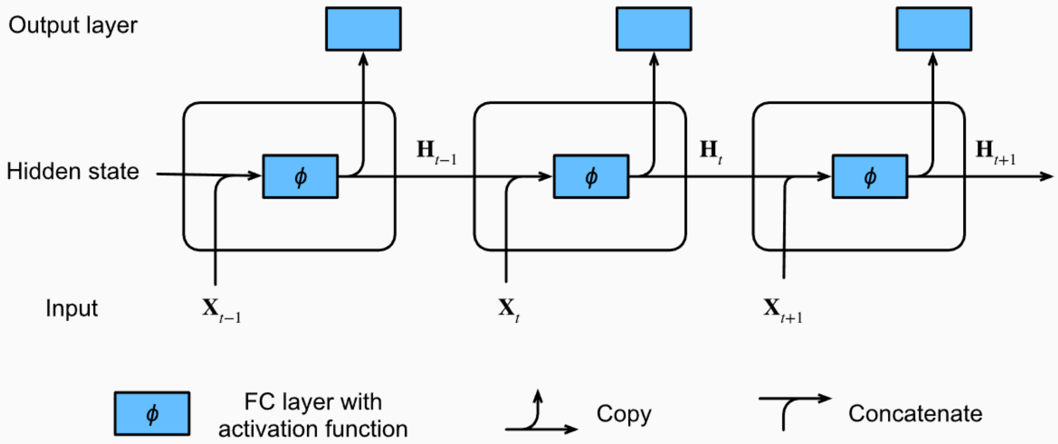
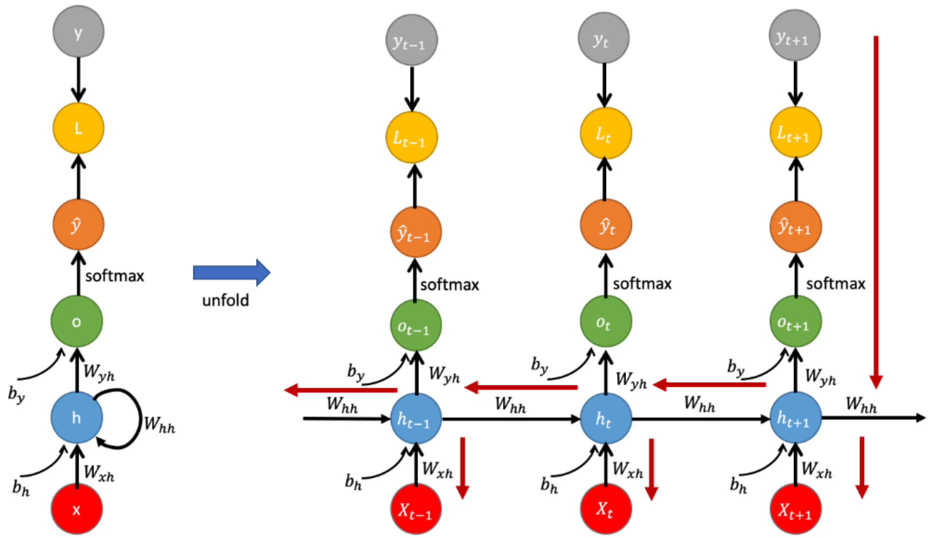


Fig. 9.4.1 An RNN with a hidden state.

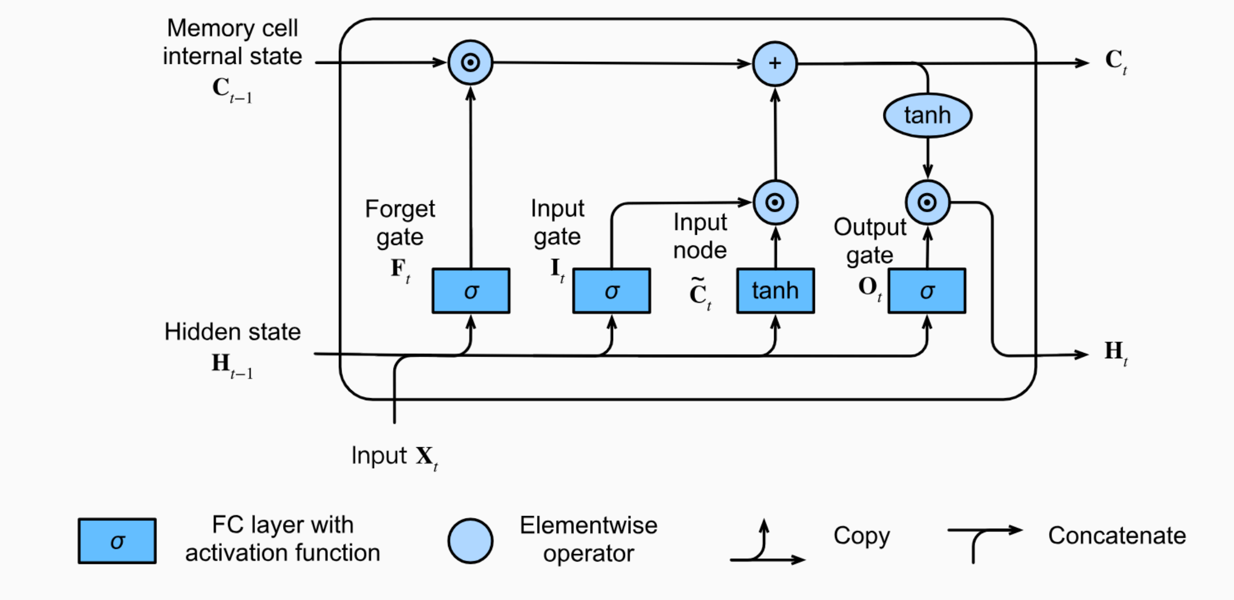
Backpropagation through time



Problems with recurrent neural networks

1. Vanishing and exploding gradient
2. Parallel training

LSTM



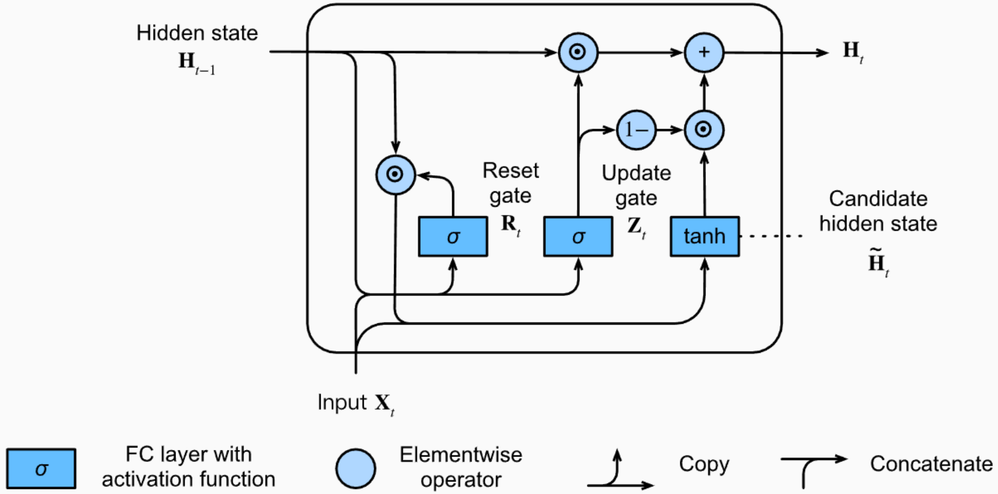
Memory cell internal state $\mathbf{C}_{t-1}$
Hidden state $\mathbf{H}_{t-1}$
Forget gate $\mathbf{F}_t$
Input gate $\mathbf{I}_t$
Input node $\tilde{\mathbf{C}}_t$
Output gate $\mathbf{O}_t$
σ
σ
tanh
σ
tanh
+
$\mathbf{C}_t$
$\mathbf{H}_t$
Input $\mathbf{X}_t$
σ
FC layer with activation function
Elementwise operator
Copy
Concatenate

LSTM (ii)

LSTM

$$
\begin{aligned}
\boldsymbol{h}_t &= \boldsymbol{o}_t \odot \tanh(\boldsymbol{c}_t) \\
\boldsymbol{o}_t &= \sigma(\mathrm{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\boldsymbol{c}_t &= \boldsymbol{f}_t \odot \boldsymbol{c}_{t-1} + \boldsymbol{i}_t \odot \tilde{\boldsymbol{c}}_t \\
\boldsymbol{f}_t &= \sigma(\mathrm{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\boldsymbol{i}_t &= \sigma(\mathrm{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\tilde{\boldsymbol{c}}_t &= \tanh(\mathrm{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}]))
\end{aligned}
$$

GRU



Hidden state
$\mathbf{H}_{t-1}$
$\mathbf{H}_t$
Reset gate
$\mathbf{R}_t$
Update gate
$\mathbf{Z}_t$
σ
σ
tanh
1−
+
Candidate hidden state
$\tilde{\mathbf{H}}_t$
Input $\mathbf{X}_t$
σ
FC layer with activation function
Elementwise operator
Copy
Concatenate

GRU (ii)

GRU

$$\boldsymbol{h}_t = (\mathbf{1} - \boldsymbol{z}_t) \odot \boldsymbol{h}_{t-1} + \boldsymbol{z}_t \odot \tilde{\boldsymbol{h}}_t$$
$$\boldsymbol{z}_t = \sigma(\mathrm{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}]))$$
$$\boldsymbol{r}_t = \sigma(\mathrm{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}]))$$
$$\tilde{\boldsymbol{h}}_t = \tanh(\mathrm{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{r}_t \odot \boldsymbol{h}_{t-1}]))$$

min-GRU

GRU

$$
\begin{aligned}
\boldsymbol{h}_t &= (\boldsymbol{1} - \boldsymbol{z}_t) \odot \boldsymbol{h}_{t-1} + \boldsymbol{z}_t \odot \tilde{\boldsymbol{h}}_t \\
\boldsymbol{z}_t &= \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\boldsymbol{r}_t &= \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\tilde{\boldsymbol{h}}_t &= \tanh(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{r}_t \odot \boldsymbol{h}_{t-1}]))
\end{aligned}
$$

$\Rightarrow$

minGRU

$$
\begin{aligned}
\boldsymbol{h}_t &= (\boldsymbol{1} - \boldsymbol{z}_t) \odot \boldsymbol{h}_{t-1} + \boldsymbol{z}_t \odot \tilde{\boldsymbol{h}}_t \\
\boldsymbol{z}_t &= \sigma(\text{Linear}_{d_h}(\boldsymbol{x}_t)) \\
\tilde{\boldsymbol{h}}_t &= \text{Linear}_{d_h}(\boldsymbol{x}_t)
\end{aligned}
$$

min-LSTM

LSTM

$$
\begin{aligned}
\boldsymbol{h}_t &= \boldsymbol{o}_t \odot \tanh(\boldsymbol{c}_t) \\
\boldsymbol{o}_t &= \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\boldsymbol{c}_t &= \boldsymbol{f}_t \odot \boldsymbol{c}_{t-1} + \boldsymbol{i}_t \odot \tilde{\boldsymbol{c}}_t \\
\boldsymbol{f}_t &= \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\boldsymbol{i}_t &= \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\tilde{\boldsymbol{c}}_t &= \tanh(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}]))
\end{aligned}
$$

$\Rightarrow$

minLSTM

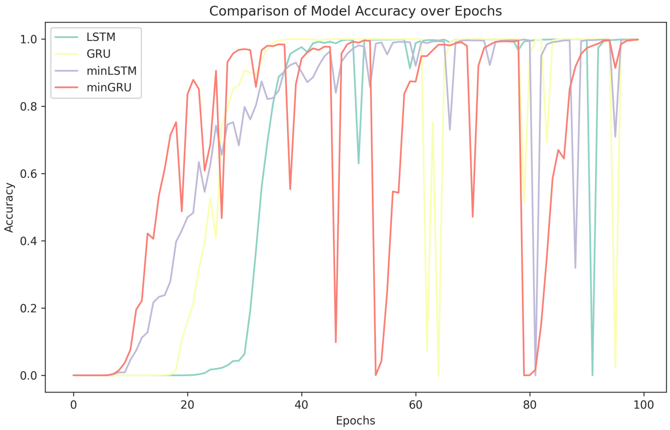
$$
\begin{aligned}
\boldsymbol{h}_t &= \boldsymbol{f}_t \odot \boldsymbol{h}_{t-1} + \boldsymbol{i}_t \odot \tilde{\boldsymbol{h}}_t \\
\boldsymbol{f}_t &= \sigma(\text{Linear}_{d_h}(\boldsymbol{x}_t)) \\
\boldsymbol{i}_t &= \sigma(\text{Linear}_{d_h}(\boldsymbol{x}_t)) \\
\tilde{\boldsymbol{h}}_t &= \text{Linear}_{d_h}(\boldsymbol{x}_t)
\end{aligned}
$$

Thesis goals

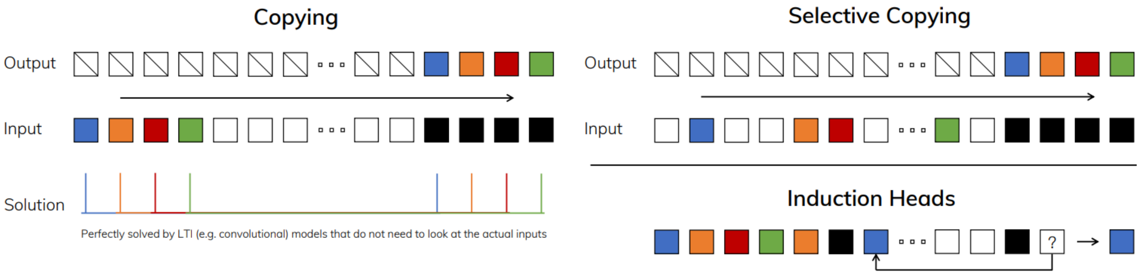
1. Implement newest models for example minLSTM, minGRU
2. Compare the models
3. Use methods Explainable AI

Work done on thesis

1. Script for generating Selective copying dataset
2. Basic comparisons of min models with originals



Selective copying



Planned work for now

1. Redo the comparisons multiple times to get more accurate readings
2. Implement scripts for better analyzing the neural networks

Zdroje

https://arxiv.org/pdf/2410.01201

https://www.d2l.ai/index.html

https://mmuratarat.github.io/2019-02-07/bptt-of-rnn

https://arxiv.org/pdf/2312.00752

https://www.researchgate.net/figure/Multi-Layer-Perceptron-MLP-diagram-with-four-hidden-layers-and-a-collection-of-single_fig1_334609713