Computational analysis of efficient gated recurrent neural networks

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Recurent neural network

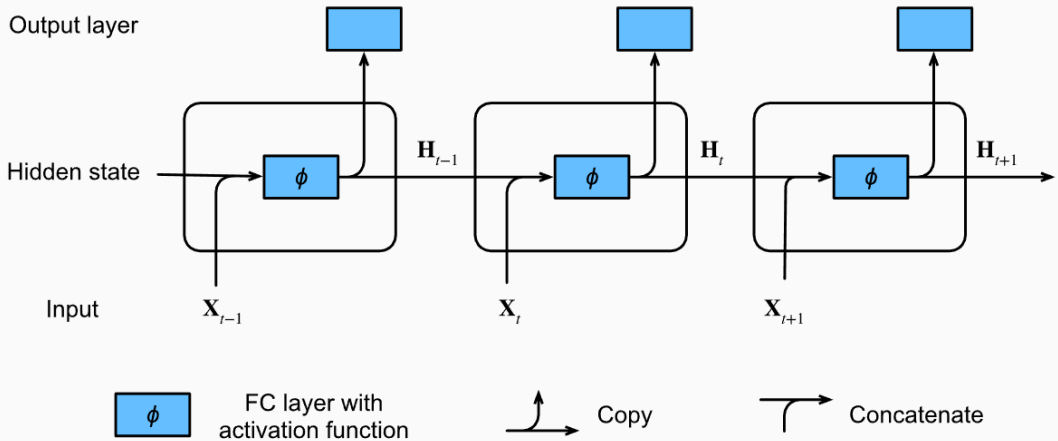
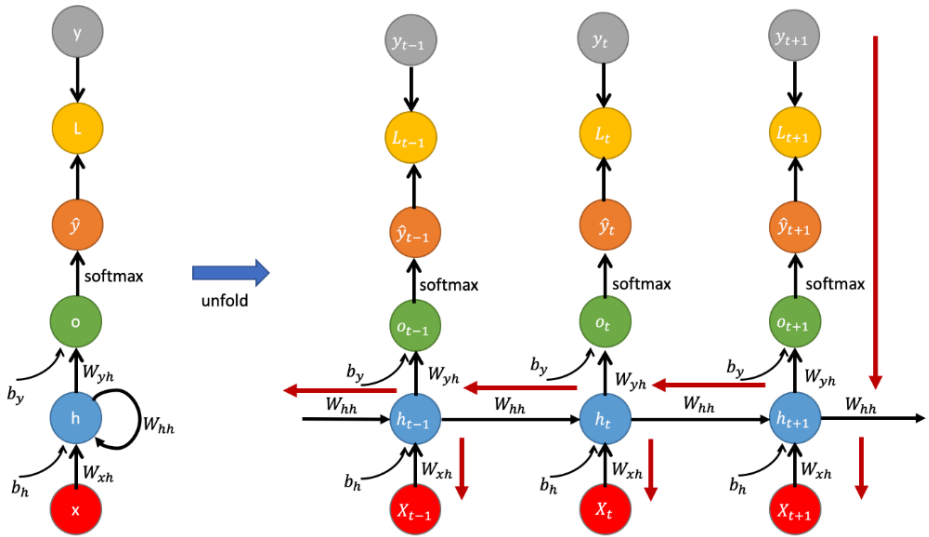


Fig. 9.4.1 An RNN with a hidden state.

Backpropagation through time



Problems with recurrent neural networks

1. Vanishing and exploding gradient
2. Parallel training

LSTM and minLSTM

LSTM

$$
\begin{aligned}
\boldsymbol{h}_t &= \boldsymbol{o}_t \odot \tanh(\boldsymbol{c}_t) \\
\boldsymbol{o}_t &= \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\boldsymbol{c}_t &= \boldsymbol{f}_t \odot \boldsymbol{c}_{t-1} + \boldsymbol{i}_t \odot \tilde{\boldsymbol{c}}_t \\
\boldsymbol{f}_t &= \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\boldsymbol{i}_t &= \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}])) \\
\tilde{\boldsymbol{c}}_t &= \tanh(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}]))
\end{aligned}
$$

$\Rightarrow$

minLSTM

$$
\begin{aligned}
\boldsymbol{h}_t &= \boldsymbol{f}_t \odot \boldsymbol{h}_{t-1} + \boldsymbol{i}_t \odot \tilde{\boldsymbol{h}}_t \\
\boldsymbol{f}_t &= \sigma(\text{Linear}_{d_h}(\boldsymbol{x}_t)) \\
\boldsymbol{i}_t &= \sigma(\text{Linear}_{d_h}(\boldsymbol{x}_t)) \\
\tilde{\boldsymbol{h}}_t &= \text{Linear}_{d_h}(\boldsymbol{x}_t)
\end{aligned}
$$

GRU and min-GRU

GRU

$$\boldsymbol{h}_t = (\mathbf{1} - \boldsymbol{z}_t) \odot \boldsymbol{h}_{t-1} + \boldsymbol{z}_t \odot \tilde{\boldsymbol{h}}_t$$
$$\boldsymbol{z}_t = \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}]))$$
$$\boldsymbol{r}_t = \sigma(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{h}_{t-1}]))$$
$$\tilde{\boldsymbol{h}}_t = \tanh(\text{Linear}_{d_h}([\boldsymbol{x}_t, \boldsymbol{r}_t \odot \boldsymbol{h}_{t-1}]))$$

$\Rightarrow$

minGRU

$$\boldsymbol{h}_t = (\mathbf{1} - \boldsymbol{z}_t) \odot \boldsymbol{h}_{t-1} + \boldsymbol{z}_t \odot \tilde{\boldsymbol{h}}_t$$
$$\boldsymbol{z}_t = \sigma(\text{Linear}_{d_h}(\boldsymbol{x}_t))$$
$$\tilde{\boldsymbol{h}}_t = \text{Linear}_{d_h}(\boldsymbol{x}_t)$$

xLSTM

1. Made of two distinct cells
2. sLSTM and mLSTM

$$c_t = \mathrm{f}_t \; c_{t-1} + \mathrm{i}_t \; z_t \qquad \text{cell state} \qquad (8)$$

$$n_t = \mathrm{f}_t \; n_{t-1} + \mathrm{i}_t \qquad \text{normalizer state} \qquad (9)$$

$$h_t = \mathrm{o}_t \; \tilde{h}_t \,, \qquad \tilde{h}_t = c_t \,/\, n_t \qquad \text{hidden state} \qquad (10)$$

$$z_t = \varphi\left(\tilde{z}_t\right) \,, \qquad \tilde{z}_t = \boldsymbol{w}_z^\top \boldsymbol{x}_t + r_z h_{t-1} + b_z \qquad \text{cell input} \qquad (11)$$

$$\mathrm{i}_t = \exp\left(\tilde{\mathrm{i}}_t\right) \,, \qquad \tilde{\mathrm{i}}_t = \boldsymbol{w}_\mathrm{i}^\top \boldsymbol{x}_t + r_\mathrm{i} \, h_{t-1} + b_\mathrm{i} \qquad \text{input gate} \qquad (12)$$

$$\mathrm{f}_t = \sigma\left(\tilde{\mathrm{f}}_t\right) \text{ OR } \exp\left(\tilde{\mathrm{f}}_t\right) \,, \qquad \tilde{\mathrm{f}}_t = \boldsymbol{w}_\mathrm{f}^\top \boldsymbol{x}_t + r_\mathrm{f} \, h_{t-1} + b_\mathrm{f} \qquad \text{forget gate} \qquad (13)$$

$$\mathrm{o}_t = \sigma\left(\tilde{\mathrm{o}}_t\right) \,, \qquad \tilde{\mathrm{o}}_t = \boldsymbol{w}_\mathrm{o}^\top \boldsymbol{x}_t + r_\mathrm{o} \, h_{t-1} + b_\mathrm{o} \qquad \text{output gate} \qquad (14)$$

xLSTM (ii)

$$\boldsymbol{C}_t = \mathrm{f}_t \; \boldsymbol{C}_{t-1} + \mathrm{i}_t \; \boldsymbol{v}_t \, \boldsymbol{k}_t^\top \qquad \text{cell state} \quad (19)$$

$$\boldsymbol{n}_t = \mathrm{f}_t \; \boldsymbol{n}_{t-1} + \mathrm{i}_t \; \boldsymbol{k}_t \qquad \text{normalizer state} \quad (20)$$

$$\boldsymbol{h}_t = \mathbf{o}_t \odot \tilde{\boldsymbol{h}}_t \,, \qquad \tilde{\boldsymbol{h}}_t = \boldsymbol{C}_t \; \boldsymbol{q}_t \; / \; \max\left\{ \left| \boldsymbol{n}_t^\top \; \boldsymbol{q}_t \right|, 1 \right\} \qquad \text{hidden state} \quad (21)$$

$$\boldsymbol{q}_t = \boldsymbol{W}_q \, \boldsymbol{x}_t + \boldsymbol{b}_q \qquad \text{query input} \quad (22)$$

$$\boldsymbol{k}_t = \frac{1}{\sqrt{d}} \boldsymbol{W}_k \, \boldsymbol{x}_t + \boldsymbol{b}_k \qquad \text{key input} \quad (23)$$

$$\boldsymbol{v}_t = \boldsymbol{W}_v \, \boldsymbol{x}_t + \boldsymbol{b}_v \qquad \text{value input} \quad (24)$$

$$\mathrm{i}_t = \exp\left(\tilde{\mathrm{i}}_t\right) \,, \qquad \tilde{\mathrm{i}}_t = \boldsymbol{w}_\mathrm{i}^\top \, \boldsymbol{x}_t + b_\mathrm{i} \qquad \text{input gate} \quad (25)$$

$$\mathrm{f}_t = \sigma\left(\tilde{\mathrm{f}}_t\right) \text{ OR } \exp\left(\tilde{\mathrm{f}}_t\right), \quad \tilde{\mathrm{f}}_t = \boldsymbol{w}_\mathrm{f}^\top \, \boldsymbol{x}_t + b_\mathrm{f} \qquad \text{forget gate} \quad (26)$$

$$\mathbf{o}_t = \sigma\left(\tilde{\mathbf{o}}_t\right) \,, \qquad \tilde{\mathbf{o}}_t = \boldsymbol{W}_\mathbf{o} \, \boldsymbol{x}_t + \boldsymbol{b}_\mathbf{o} \qquad \text{output gate} \quad (27)$$

Thesis goals

1. Compare the models
2. Use methods Explainable AI

Work done on thesis

1. Prepared scripts and workflows for mass training
2. Done testing on timeseries datasets

Zdroje

https://arxiv.org/pdf/2410.01201

https://www.d2l.ai/index.html

https://mmuratarat.github.io/2019-02-07/bptt-of-rnn

https://arxiv.org/pdf/2312.00752

https://www.researchgate.net/figure/Multi-Layer-Perceptron-MLP-diagram-with-four-hidden-layers-and-a-collection-of-single_fig1_334609713

https://arxiv.org/pdf/2405.04517